Title
Essays on Macroeconomics

Author
Jang, Jaeyoung

Publication Date
2020

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA

Los Angeles

Essays on Macroeconomics

A dissertation submitted in partial satisfaction

of the requirements for the degree

Doctor of Philosophy in Economics

by

Jaeyoung Jang

2020

ABSTRACT OF THE DISSERTATION

Essays on Macroeconomics

by

Jaeyoung Jang
Doctor of Philosophy in Economics
University of California, Los Angeles, 2020
Professor Lee Ohanian, Chair

These essays contribute to the literature on Macroeconomics. Chapter 1 provides an endogenous growth model to explain the impact of financial reform on a nation's growth rate both in the short run and the long run. The current literature focuses only on the transition period right after the reform and therefore, cannot match the long-run growth rate that is higher than the pre-reform growth rate. The model primarily operates through a borrowing constraint on firms that prevents them from optimally allocating resources between capital investment and innovation. Through this mechanism, it can explain all three phases of a country's growth around a financial reform: A very low growth period before the reform, rapid growth immediately following the reform, and finally a convergence to a moderate growth rate in the long run.

Chapter 2 develops a monopolistic competition model with multi-sector network linkages. In the presence of monopolistic competition, information of firm size is not sufficient information to measure an individual firm's impact on the economy. Therefore, the interaction between firms should be considered to measure the impact properly.

Chapter 3 inspects the impact of input-output linkages on gains from trade. I extend the model from Chapter 2 to an open economy. The conventional issue in the current literature is that the welfare gain from trade is too small. The model is different from existing models with input-output linkages in that it cannot only compare welfare gains with standard

models, but also enables a counter-factual analysis to examine the importance of network linkages by shutting down the relationship across sectors within the country. In the model, opening trade delivers newly introduced goods to a firm in the country. These newly traded goods will be used to produce other goods in a more efficient way. Through this channel, measured gains from trade are bigger than in the standard literature. The input-output linkages initiate a positive chain reaction through the economy and produce an additional channel for welfare gain that is absent in standard models, thereby increasing measured welfare gain.

The dissertation of Jaeyoung Jang is approved.

Gary D. Hansen

Saki Bigio

Mark J. Garmaise

Lee Ohanian, Committee Chair

University of California, Los Angeles

2020

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGMENTS

I am deeply indebted to my advisor Lee Ohanian, Gary Hansen, Saki Bigio and Mark Garmaise for guidance and great advise. I would also thank all participants at the proseminar of UCLA. Also, I would like to thank my parents and wife for their care and love.

VITA

2014	B.A. Yonsei University Seoul, Korea
2016	C.Phil., Economics University of California, Los Angeles Los Angeles, California

CHAPTER 1

Finance, Misallocation and Growth: Quantifying the Contribution of Financial Market Imperfections and Reforms to East Asian Growth Miracles

1.1 Introduction

Starting from Schumpeter (1911)'s argument that a sound financial system improves a nation's wealth, the importance of the financial system has been widely emphasized in many countries' economic policies as well as the academic literature. Schumpeter argues a well developed banking system promotes innovation activity and induces economic growth. This paper develops a model that can help us understand the connection between finance and innovation through the lens of the interplay between investment in innovation and inefficient allocation of resources due to capital market imperfection. In addition, we match the historic features of Asian countries' rapid growth after financial reforms - 1) extremely high growth immediately following reforms, 2) persistent high growth for several decades, 3) very high investment to output ratios, and 4) finally and most importantly, higher steady state growth rate than pre-reform steady state growth rate after settling down.

We develop an endogenous growth model to study the impact of financial market imperfections on economic growth through capital misallocation. Previous research on misallocation mainly focuses on the level of output, but often fails to emphasize the long run growth effect of the misallocation. Papers such as Mino (2015) and Laeven et al. (2015) study the long run growth effect of the capital market imperfection, but don't explore transitional dynamics. In contrast, our model can capture the full picture of financial reform on economic growth. Following Buera and Shin (2013) and Moll (2014), we introduce collateral constraint as a form of capital market imperfection. Capital serves not only as an input, but also as collateral to facilitate access to funds for production. This collateral constraint results in over-accumulation of capital, which deters the economy from investing in innovation. Finally, the imbalance between capital and human capital as accumulative assets lowers aggregate efficiency thereby lowering economic growth.

Our model can capture the salient features of the Asian miracle after financial reform. The outstanding postwar growth of Asian countries has attracted many researchers' attention. Young (1995) and Klenow and Rodríguez-Clare (1997) argue that capital and human capital accumulation are the main causes of the rapid growth. On the other hand, Easterly and Levine (2001) point out the overestimation problem in their work and attribute the increase in total factor productivity (TFP) to the rapid growth. To reconcile these two opposing views, my model focuses on the role of financial constraints. When firms are financially constrained, the more efficient firms don't always have access to capital lowering aggregate TFP and holding down investment in human capital. When the constraints are lifted, TFP and investment in physical and human capital increase simultaneously.

Building on the early contributions of the two pioneering papers discussed above, there is a large literature studying the Asian growth miracle. Prescott and Parente (1994), Pack and Nelson (1997), Song et al. (2011) and Buera and Shin (2013) are notable theoretical papers that explain the extraordinary growth of Asian countries. They study the cause of increase in total factor productivity through various channels (e.g finance, trade, and technology barrier). On the empirical side, Stiglitz and Yusuf (2001), Gourinchas and Jeanne (2013) and López (2005) summarize the potential cause and dynamics of the East Asian miracle.

This strand of literature mainly focuses on the rapid and persistent growth of Asian countries during the miracle itself, but is comparatively silent regarding the post-miracle trends. Figure 1.1 describes the pattern of the so-called Asian miracle after the reforms. These countries' average annual growth rates of GDP per capita were around zero in 1950s. In the beginning of 1960s, Hong Kong, South Korea, and Taiwan all conducted enormous financial reforms that spurred the growth rates up to 7%.[1]. They experienced persistent rapid growth through the 1970s and 1980s, but the growth rates have been decelerating thereafter. Eventually, the growth rate of GDP per capita converges from 2% to 3% , which is higher than the pre-reform growth rate of 0%. This motivates us to develop an endogenous growth model to exhibit a permanent-growth effect from financial reforms.

Financial reform is closely related to misallocation because when there are capital market imperfections, resources are not efficiently allocated to their best uses - the more efficient firms cannot use as much capital as they want. Recent work has actively studied the relationship between misallocation and aggregate inefficiency. Hsieh and Klenow (2009), Restuccia and Rogerson (2008) and Midrigan and Xu (2014) explain how resource misallocation can affect aggregate total factor productivity. Buera and Shin (2013) and Moll (2014) study the impact of financial reform in both the short-term and the long-term. These papers, however, only focus on the level of output. Therefore, they cannot comment on the effects of financial reform on the long-run growth rate, which motivates the use of an edogenous growth framework to fill this gap.

There is also a notable endogenous growth literature with capital market imperfection. Early contributions include Boyd and Prescott (1986), Newman and Banerjee (1993), King and Levine (1993) and Aghion and Bolton (1997). They assume a representative agent model to study the impact of financial frictions on the long-run growth rate, which is limited in performing richer analyses of misallocation. Jones (2011), Mino (2015), Laeven et al. (2015), and Anzoategui et al. (2019) are recent papers that utilize the heterogeneous agent setup. All of them provide excellent insights into the contribution of financial reform to a nation's long-run growth rate. We contribute to this literature by developing a tractable model with the transitional dynamics that they currently miss. The study of transitional dynamics is crucial in understanding the entire feature of the Asian Miracle, both qualitatively and quantitatively.

In section 2, we develop an endogenous growth model with collateral constraints. In

[1] Details of 1960's financial reforms are described in the Appendix

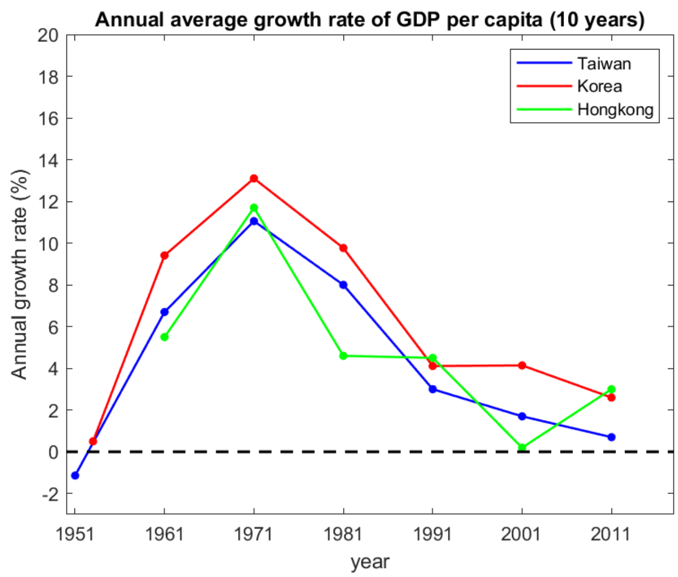


Figure 1.1: Annual average growth rate of Asian countries

section 3, we will study the long-run effect of financial reform. Finally, in section 4, we will see transitional dynamics to match the salient features of Asian miracle.

1.2 The Model

We construct a continuous time endogenous growth model to study the role of financial market imperfection in economic growth. The economy grows as firms invest in physical and human capital. We can think of these two kinds of investment as differing in a few important ways. First, there will be a market for physical capital while human capital will be specific to each firm. Second, investment in physical capital gives a guaranteed return while investment in human capital is risky. Finally, we will introduce a borrowing constraint that depends on a firm's current stock of physical capital - they can only borrow up to a percentage of their currently owned assets. This last assumption generates a situation where firms need to overinvest in physical capital in order to avoid hitting their constraint, which will imply a lower growth rate than in an unconstrained model.

1.2.1 Simple explanation of the model mechanism

The model is closely related to Buera and Shin (2013) and Moll (2014) by using a collateral constraint as the primary market imperfection. These models, as with most papers that study misalloacation take the following aggregate production form:

$$T(\psi)K^{\alpha}L^{1-\alpha}. \tag{1.1}$$

In this model, $T(\psi)$ is the total factor productivity representing the economy's aggregate efficiency, K the aggregate capital, and L the aggregate labor. Suppose ψ is the degree of misallocation caused by capital market imperfection. In this case, GDP per capita at the steady state depends only on ψ. Severe misallocation (High ψ) results in low aggregate productivity (low T), and vice versa. With this form, reallocating resources from inefficient firms to more efficient ones will result in static gains. However, undoing the misallocation would not have a permanent effect on growth rates on its own. To address this concern, we take an approach with an AK structure, which will result in a different long-run GDP per capita growth rate. The reduced form of our model in this case looks like:

$$T(\psi)K. \tag{1.2}$$

Following Romer (1990), the growth rate at the steady state is $T(\psi) - \rho$ where ρ is a discount rate. As a result, the capital market imperfection indirectly affects the long run growth rate through misallocation. To generate misallocation, we assume each individual producer has a production function of the form

$$Az^{\alpha}k^{1-\alpha}. \tag{1.3}$$

A is the productivity level, which is identical across agents and z represents human capital that can be accumulated only through investment in innovation. This production still satisfies AK structure because both capital and human capital are accumulative assets. Collateral constraints will motivate agents to accumulate excess physical capital, which diverts resources from innovation and lowers aggregate inefficiency.

1.2.2 Preference and Technology

There are a continuum of entrepreneurs with preferences given by

$$E_0 \int_0^\infty e^{-\rho t} \ log \ c_t \ dt \tag{1.4}$$

where ρ is a discount rate. Each agent is endowed with their own productivity (z) and wealth (a). Combining their productivity and physical capital k they produce output according to

$$Az_t^\alpha k_t^{1-\alpha}. \tag{1.5}$$

A represents total factor productivity which is identical across agent. α is human capital share. (1-α) is capital share.[2]

1.2.3 Collateral constraint and budget constraint

Physical capital is the only tradable asset in this economy. Entrepreneurs can rent (lend) physical capital $k - a$ ($a - k$) from the capital market at a interest rate r_t. However, they are restricted in borrowing capital according to the following constraint.

$$k_t \leq \mu a_t \tag{1.6}$$

The collateral constraint parameter is denoted by $\mu \geq 1$. $\mu = 1$ implies financial autarky - firms can only use capital they already own. In contrast, $\mu = \infty$ means a friction-less economy where firms have no limits on borrowing. After production, entrepreneurs make

[2] We can generalize the model to incorporate labor where each agent can use L units of his own labor. In this case, we can rewrite down the production function with labor-augmenting human capital $(A(zL)^\alpha k^{1-\alpha}) = (\tilde{A}(z)^\alpha k^{1-\alpha})$ where $\tilde{A} = AL^\alpha$

a decision to distribute their output along three margins - consumption, c, investment in physical asset, $\dot{a}$ and investment in innovation, x. Therefore, the budget constraint follows

$$\dot{a}_t = \underbrace{Az_t^\alpha k_t^{1-\alpha} - r_t(k_t - a_t)}_{\text{profit}} - c_t - x_t \tag{1.7}$$

In words, Equation (1.7) says that the change in total assets is profit (production minus cost of borrowing) minus consumption and investment in innovation.

The production decision is a purely static problem. Therefore, we can analytically distinguish financially constrained firms and financially non-constrained firms. So the budget constraint can be simplified as

$$\begin{aligned} \dot{a}_t = & Az_t^\alpha(\mu a_t)^{1-\alpha} - (\mu - 1)r_t a_t - c_t - x_t \quad \text{if} \quad \frac{a_t}{z_t} \leq \frac{1}{\mu}(\frac{(1-\alpha)A}{r_t})^{\frac{1}{\alpha}} \\ & A^{\frac{1}{\alpha}}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} r_t^{-\frac{1-\alpha}{\alpha}} z_t + r_t a_t - c_t - x_t \quad \text{if} \quad \frac{a_t}{z_t} > \frac{1}{\mu}(\frac{(1-\alpha)A}{r_t})^{\frac{1}{\alpha}}. \end{aligned} \tag{1.8}$$

One important distinction between this setup and similar papers that use a financial constraint is that small firms (in terms of assets) are not necessarily constrained. Whether they are financially constrained only depends on the ratio of assets to productivity, $\frac{a_t}{z_t}$. An entrepreneur who has relatively better productivity is more likely to rent more capital, and thus has more chance to be financially constrained unless he has enough assets to leverage.

1.2.4 Innovation flow

The basic structure of innovation follows the standard quality ladder introduced in previous work from Grossman and Helpman (1991) and Acemoglu and Cao (2015)

$$z_t(\xi) = \lambda^n z_s(\xi) \tag{1.9}$$

where n is a random variable that represents the number of innovation success between time s and t. The equation shows the number of successful innovations agent ξ received between time s and t, $s \leq t$. λ is the jump size variable meaning how much they can improve the productivity from innovation.

An entrepreneur can influence the probability of a successful increase in productivity by investing in human capital. In particular, an entrepreneur can spend x_t units of goods to generate an innovation flow rate $\theta \frac{x_t}{z_t}$. θ is an intensity parameter representing the success rate of innovation. As they spend more resources in innovation, x_t, they have a better chance to get higher productivity. Specifically, the innovations follow a Poisson process. When a company invests x_t units of goods in innovation, the probability of the number of success during the short period of time, Δt is

$$P(N(\Delta t) = n) = \frac{(\theta \frac{x_t}{z_t} \Delta t)^n}{n!} e^{-\theta \frac{x_t}{z_t} \Delta t}. \tag{1.10}$$

As entrepreneurs invest more resource in innovation, the innovation success rate will increase. On the other hand, as the current productivity enhances, they should invest more resources in innovation to keep the same R&D success rate.

1.2.5 Individual behavior and HJB equation

Given the path of the equilibrium interest rate r_t, individuals maximize (1.4) subject to (6) and (7), facing the endogenous innovation Poisson intensity (1.9) and (1.10). A Hamilton-Jacobi-Bellman (HJB) equation for the individual maximization problem at time t takes the following form[3]

$$\begin{aligned} \rho V_t(a, z) - \dot{V}_t(a, z) = \max_{c,k} log\, c &+ \frac{\partial V_t}{\partial a}(A z^{\alpha} k^{1-\alpha} + r_t(a - k) - c - x) \\ &+ \theta \frac{x}{z}(V_t(\lambda z, a) - V_t(z, a)) \\ &+ \psi_t(\mu a - k) \end{aligned} \tag{1.11}$$

[3]Detailed derivation is in Appendix

subject to

$$\psi_t(\mu a - k) = 0, \psi_t \geq 0 \tag{1.12}$$

The intuition of equation (1.11) is straightforward. The flow rate of the present discounted value function is equal to the instantaneous utility function plus the marginal utility gain from saving capital plus the marginal utility gain from innovation. Equation (1.12) gives a complementary slackness condition.

1.2.6 Equilibrium

An equilibrium in this economy is time paths for the distribution of $\Omega_t(z, a)$ and policy functions $k_t(z, a), x_t(z, a), c_t(z, a)$ that maximize (1.26) given the path of r_t, and satisfy:

- Capital market clearing condition

$$\int k_t(z, a) \;\; d\Omega_t(z, a) = \int a \;\; d\Omega_t(z, a) \tag{1.13}$$

- Law of motion of distribution (ω is density of Ω)

$$\frac{\partial \omega_t(z, a)}{\partial t} = \underbrace{-\frac{\partial}{\partial a}(\dot{a}(z, a)\omega_t(z, a)) - \theta \frac{x_t(z, a)}{z}\omega_t(z, a)}_{\text{Outflow}} + \underbrace{\theta \frac{x_t(\lambda^{-1}z, a)}{\lambda^{-1}z}\omega_t(\lambda^{-1}z, a)}_{\text{Inflow}} \tag{1.14}$$

Physical capital is the only tradable asset in this economy, therefore, we only have one market clearing condition. Equation (1.14) is the conventional Kolmogorov Forward Equation (KFE) illustrating the movement of the distribution. The mass at each state (a, z) depends on 2 margins: investment in capital and investment in innovation. As entrepreneurs invest more, they have greater chances of getting out of the current state. Finally, inflow into a state comes from agents who are technologically one step behind.

1.3 Stationary Equilibrium

In this section, we will consider a stationary equilibrium to study the long-run growth effect of capital misallocation. In a friction-less economy, there is an optimal capital to human capital ratio because the complementary effect between capital and human capital drives the economy to keep a balance between them.

In the economy with collateral constraints, capital serves not only as an input, but also as a collateral to give access to funds for production. Therefore, constrained entrepreneurs try to overcome financial constraints by investing in capital. This self-financing motivation temporarily alleviates the misallocation in an individual level.[4] However, entrepreneurs tend to accumulate capital which they otherwise would have invested in innovation thereby reducing the aggregate efficiency.

1.3.1 Friction-less Economy

Let us start with balanced growth path (BGP) of the friction-less economy.

Theorem 1. *(BGP in a friction-less economy) A BGP of the economy is an equilibrium path where aggregate productivity level, $\bar{z}_t$, and aggregate capital $\bar{a}_t$ grow at a constant rate g and the interest rate r_t is constant*

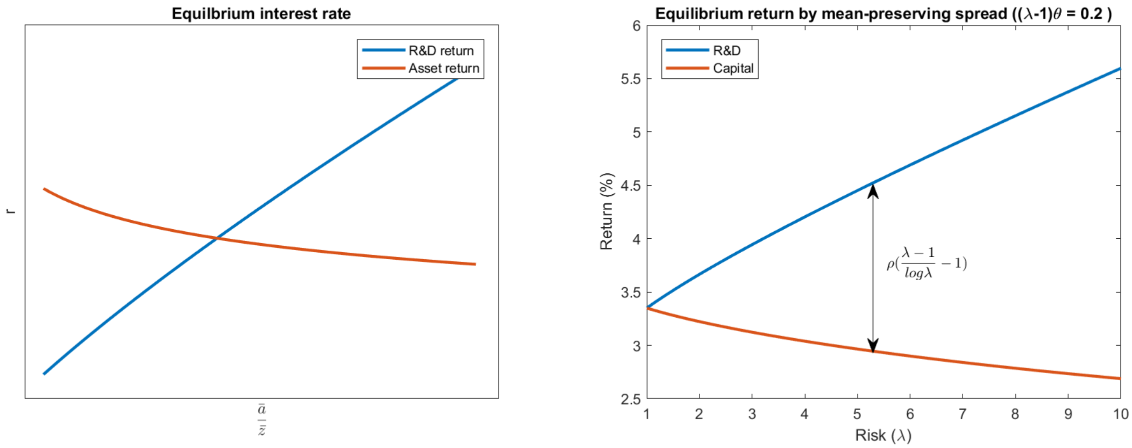
$$r = (1-\alpha)A(\frac{\bar{a}}{\bar{z}})^{-\alpha} \tag{1.15}$$

$$g = r - \rho \tag{1.16}$$

where $\frac{\bar{a}}{\bar{z}}$ is a solution for the following nonlinear equation

$$\underbrace{\theta(\lambda-1)\alpha A\Big(\frac{\bar{a}}{\bar{z}}\Big)^{1-\alpha} + \rho - \frac{\rho(\lambda-1)}{log\lambda}}_{\textit{Risk-adjusted return from R\&D}} = \underbrace{(1-\alpha)A\Big(\frac{\bar{a}}{\bar{z}}\Big)^{-\alpha}}_{\textit{Return from physical asset}} \tag{1.17}$$

[4] This effect is well illustrated in $Moll$ (2014).



(a) Return on assets v.s R&D

(b) Return Spread

Figure 1.2: Return from asset and Return spread by risk

Proof. See the Appendix □

Equation (1.38) characterizes the optimal aggregate capital to human capital ratio.[5] Figure 1.2a shows the risk-adjusted return from innovation and the return from capital. Because each asset exhibits diminishing returns, the return from capital (innovation) will decrease as $\frac{\bar{a}}{\bar{z}}$ increases (decreases). Therefore, the optimal ratio is determined where the risk-adjusted innovation return and the capital return meet each other.

Capital asset is a safe asset, but human capital is risky. Therefore, there should be a risk premium for innovation, $\frac{\rho(\lambda-1)}{log\lambda} - \rho$. Holding constant the expected return from innovation, $\theta(\lambda-1)$, Figure 1.2*b* shows that the risk premium increases as the risk of innovation increases. On the other hand, the risk premium collapses to 0 as $\lambda \to 1$. In this case, investment in innovation is the same as investing in capital because there is no uncertainty. Finally, the the return from capital (1.36) equals the aggregate marginal productivity of capital.

Relation to AK model and quality ladder model

Through Equations (1.36) to (1.38), we can get the following approximation when λ is around

[5]We can approximate the optimal asset to human capital ratio $\left(\frac{\bar{a}}{\bar{z}}\right)^{\text{optimal}} \approx \frac{1-\alpha}{\alpha}\frac{1}{\theta(\lambda-1)}$ when λ is around 1. The optimal ratio can be different depending on parameters. For example, the ratio will be low as the human capital share is lower and innovation technology parameters, θ and λ, are higher

1.

$$g \approx \alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\{\theta(\lambda-1)\}^{\alpha}A-\rho, \tag{1.18}$$

We can easily see the model follows the simple AK model in the limiting case where human capital share, α, is 0. On the other hand, the model collapses into the quality ladder model when α goes to 1.

Unlike the standard misallocation literature, the production function is linear in accumulative assets - capital and human capital. Therefore, the production function is similar to an AK structure.[6] Missing one feature or another makes the economy converge to the steady state output, which implies that countries' long-run growth rate will be identical without exogenously imposing growth rate.

1.3.2 Economy with Financial Friction

In this section, we will show how financial frictions generate inefficiency, thereby lower long-run growth rate. We will see that inefficiency arises as firms overinvest in physical capital to get around the constraint. Since solving this two dimensional problem is challenging, let us start with changing 2-dimensional problem to a 1-dimensional problem where the only state variable is $\tilde{a}=\frac{a}{z}$. Another benefit of this approach is that we can easily get a stationary distribution of $\tilde{a}$.

Theorem 2. *(Stationarized HJB equation - Modified Gibrat's law)*
When $r > \rho$, individual behavior only depends on $\tilde{a} = \frac{a}{z}$ and $V(a,z) = v(\frac{a}{z}) - \frac{1}{\rho}\ log\ z$ where $v(\tilde{a})$ follows

$$\begin{aligned}\rho v(\tilde{a}) = \max_{\tilde{c},\tilde{k},\tilde{x}} log\ \tilde{c} &+ \frac{\partial v}{\partial \tilde{a}}(A\tilde{k}^{1-\alpha} - r\tilde{k} + r\tilde{a} - \tilde{c} - \tilde{x}) \\ &+ \theta\tilde{x}(v(\frac{\tilde{a}}{\lambda}) - v(\tilde{a}) + \frac{1}{\rho}log(\lambda)) \\ &+ \tilde{\psi}(\tilde{a})(\mu\tilde{a} - \tilde{k})\end{aligned} \tag{1.19}$$

[6] The simplest example of AK structure with two accumulative asset is $Ak_1^{\alpha}k_2^{1-\alpha}$

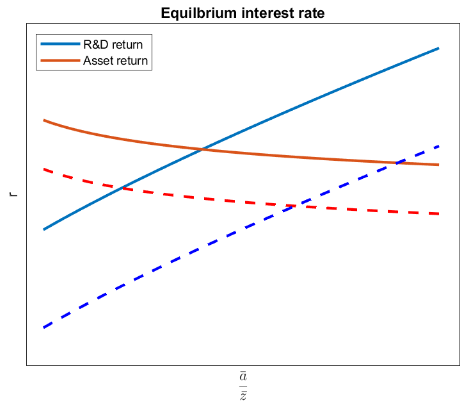


Figure 1.3: Return on assets v.s R&D in friction economy

Proof. See the Appendix □

Equation (1.19) tells us that entrepreneurs' behavior only depends on the ratio of capital to human capital, $\tilde{a} \equiv \frac{a}{z}$. More precisely, all the decisions will be scaled by the level of human capital conditional on $\tilde{a}$. This result implies firms will randomly grow independent of size conditional on $\tilde{a}$ (Gibrat's law).

In economy with collateral constraints, the internal cost of borrowing is $r + \frac{\psi(\tilde{a})}{\frac{\partial v}{\partial \tilde{a}}}$, and the gain from accumulating assets is $r + \frac{\mu\psi(\tilde{a})}{\frac{\partial v}{\partial \tilde{a}}}$. Therefore, capital is more attractive compared to friction-less economy. On the other hand, the innovation is relatively less attractive because the higher internal cost of borrowing reduces the profit from business. As we see Figure 1.3, the return from innovation will be reduced because of aggregate inefficiency, resulting in a higher capital to human capital ratio and lower equilibrium interest rate. In sum, the self-financing motivation diverts the resources from innovation.

The output decision is still static even in the stationarized HJB equation. Because of linearity of return from investment, entrepreneurs invest all resources in either capital or innovation. Therefore, we can classify the entrepreneurs behavior into 4 stages.

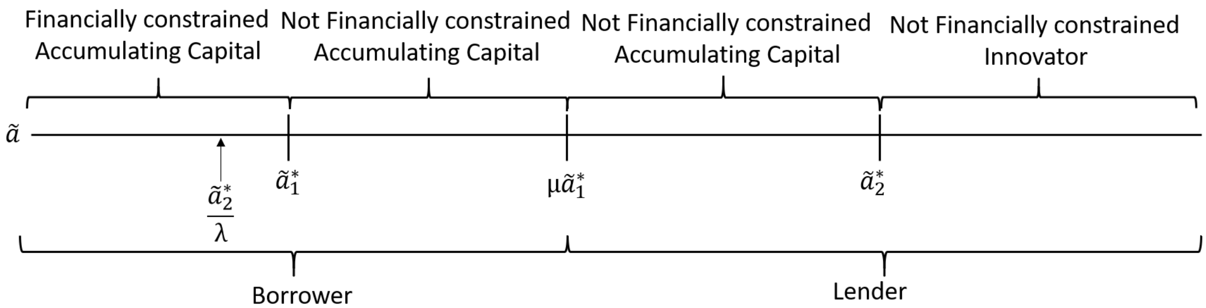


Figure 1.4: 4 stages of entrepreneur

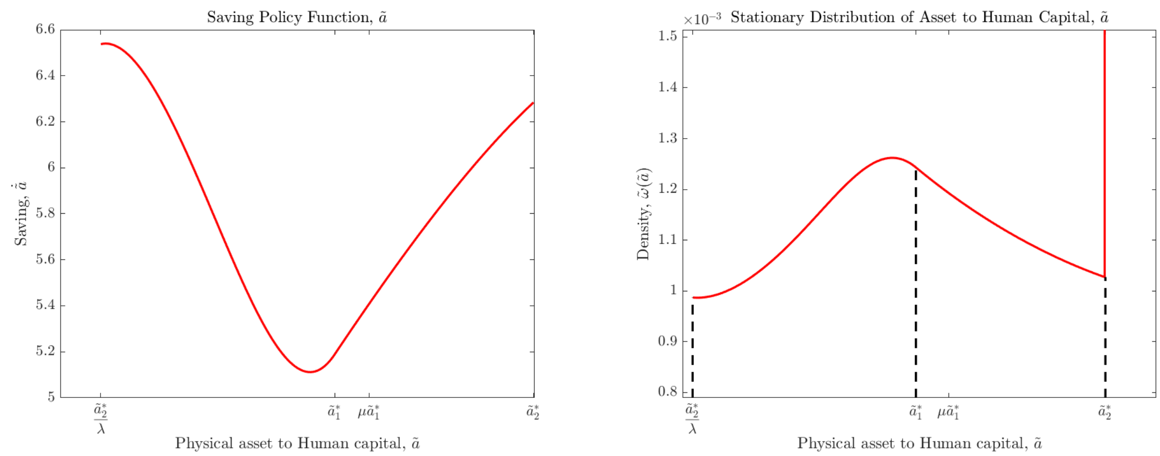
Corollary 1. *There exists* $\tilde{a}_1^* \equiv \frac{1}{\mu}(\frac{(1-\alpha)A}{r})^{\frac{1}{\alpha}}$, $\tilde{a}_2^* \in \mathbb{R}^+$ *and* $\tilde{a}_2^* > \tilde{a}_1^*$

$$\begin{aligned}\rho v(\tilde{a}) = &\max_{\tilde{c}} log\ \tilde{c} + \frac{\partial v}{\partial \tilde{a}}(A\mu^{1-\alpha}\tilde{a}^{1-\alpha} - (\mu-1)r\tilde{a} - \tilde{c})\ if\ \tilde{a} \leq \tilde{a}_1^* \\ &\max_{\tilde{c}} log\ \tilde{c} + \frac{\partial v}{\partial \tilde{a}}(\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}A^{\frac{1}{\alpha}}r^{-\frac{1-\alpha}{\alpha}} + r\tilde{a} - \tilde{c})\ \ if\ \tilde{a}_1^* < \tilde{a} \leq \tilde{a}_2^* \\ &\max_{x} log\ (\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}A^{\frac{1}{\alpha}}r^{-\frac{1-\alpha}{\alpha}} + r\tilde{a} - \tilde{x}) + \theta\tilde{x}(v(\frac{\tilde{a}}{\lambda}) - \tilde{v}(\tilde{a}) + \frac{1}{\rho}log(\lambda))\ \ if\ \tilde{a}_2^* < \tilde{a}\end{aligned} \tag{1.20}$$

Proof. See the Appendix □

The figure 1.4 illustrates 4 stages of entrepreneurs' capital accumulation behavior. Below $\tilde{a}_1^*$, entrepreneurs are financially constrained and their collateral constraint will be binding. Moreover, we can find an analytic solution to the threshold given the interest rate, r. Entrepreneurs between $\tilde{a}_1^*$ and $\mu\tilde{a}_1^*$ can borrow as much capital as they need - they are no longer financially constrained. Above $\mu\tilde{a}_1^*$, they start to lend some of their capital because they have too much capital compared to their technology. Therefore, the gain from interest will be eventually higher than the marginal gain from production. At $\tilde{a}_2^*$, they begin to invest in innovation.[7] They start to invest in innovation as soon as they reach the threshold point. If they fail to succeed in innovation, they will stay at the threshold and keep researching, but if they succeed, their endowed human capital level will jump by λ. Success in innovation has

[7] The constraint threshold point $\tilde{a}_1^*$ is always less than innovation because marginal gain from asset goes to infinite $\frac{\partial v}{\partial \tilde{a}} \to \infty$, as $\tilde{a} \to 0$. However, this is not true in the case of friction-less market because the marginal gain from asset is r.



(a) Policy function

(b) Stationary distribution

Figure 1.5: The example of Saving policy function and Stationary distribution

two trade-offs: (1) The overall productivity level will enhance the efficiency of production, thereby increasing profit. (2) The capital to human capital ratio will decrease, so the firm will become financially constrained.

The interesting point here is that entrepreneurs keep accumulating capital even after they become lenders because of a precautionary saving motive - innovating sooner means that it takes more time to be non-financially constrained. Therefore, they have an incentive to accumulate capital and enjoy the current interest for a while.

Definition 1.3.1. *(Stationary BGP)*
A stationary BGP of this economy is a equilibrium path where the distribution of $\tilde{\Omega}(\tilde{a})$ and policy functions $\tilde{k}(\tilde{a}), \tilde{x}(\tilde{a}), \tilde{c}(\tilde{a})$ that maximize the stationarized HJB equation (1.20) given r and satisfy:

- *Capital market clearing condition*

$$\int \tilde{k}(\tilde{a})d\tilde{\Omega}(\tilde{a}) = \int \tilde{a}\ d\tilde{\Omega}(\tilde{a}) \tag{1.21}$$

- *Stationary distribution ($\tilde{\omega}$ is density of $\tilde{\Omega}$)*

$$0 = \underbrace{-\frac{\partial}{\partial \tilde{a}}(\dot{\tilde{a}}(\tilde{a})\tilde{\omega}(\tilde{a})) - \theta \tilde{x}(\tilde{a})\omega(\tilde{a})}_{\textit{Outflow}} + \underbrace{\theta \tilde{x}(\lambda \tilde{a})\tilde{\omega}(\lambda \tilde{a})}_{\textit{Inflow}} \tag{1.22}$$

and the growth rate of aggregate consumption, capital and human capital is $g = r - \rho$.

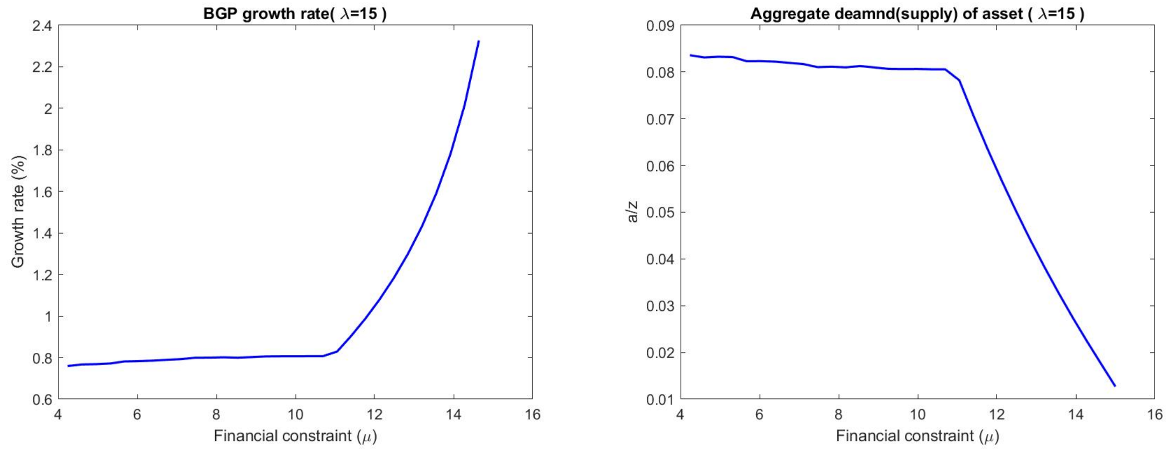
In general, the stationary BGP doesn't have an analytic solution; thus, we should solve it computationally. Based on Achdou et al. (2017), we use a finite difference method to solve the HJB equation and KFE. Furthermore, we extend their method to get the endogenous innovation threshold point, $\tilde{a}_2$, where the entrepreneurs feel indifferent between investing in either assets. The stationary distribution looks similar to the wealth distribution in a continuous time version of Huggett (1993) studied again by Achdou et al. (2017). Finally, we use the algorithm similar to Aiyagari (1994) to find the equilibrium interest rate.

Figure 1.5*b* describes the general shape of the stationary distribution. The stationary distribution is the convolution of continuous and discrete distributions and features a Dirac mass at the innovation threshold point because innovator only has small chance to get a new technology (intuitively a bottle neck effect). Contrary to the conventional Hugget-Bewley model and Achdou et al. (2017), the boundary is endogenous as the result of maximization behavior. The stationary distribution is closely related to the stages of entrepreneurs. Because there is only positive drift when they are accumulating capital and success in innovation will decrease the ratio by λ, the mass should be between $\frac{\tilde{a}_2^*}{\lambda}$ and $\tilde{a}_2^*$. This feature is closely related to the following property.

Proposition 1. *If $\mu > \lambda$, $g = g^{optimal}$ where $g^{optimal}$ is the friction-less aggregate growth rate.*

Proof. See the Appendix □

Proposition 1 implies that impact of the capital market imperfection is asymmetric between underdeveloped countries and developed countries. For financially developed countries



(a) Long-run growth rate

(b) Capital to Human capital

Figure 1.6: Impact of financial market imperfection

(high μ), additional financial reform would not have any further benefit because precautionary self-financing is enough to cover the collateral constraint. On the other hand, the financial reform will lead to a permanent effect on long-run growth rate for financially underdeveloped countries (low μ). This proposition is consistent with the history of financial reforms in South Korea. There is a consensus that the 1960s financial reforms had a significant effect on growth miracle in South Korea. The government's main goal in 1960 was to support promising companies as a means of export-oriented growth, which included several financial reforms.[8]. As a result, the annual growth rates were 9% and 13% in 1960's and 1970's, respectively. On the other hand, there was another large-scale financial reform after the 1997 Asian financial crisis. The IMF-led financial reform was beneficial for South Korea to recover between 1998 and 2000, but they did not see the growth miracle as they experienced before because their financial system was already relatively developed.

Figure 1.6a and Figure 1.6b shows the relationship between financial development and long-run growth rate, and the equilibrium capital to human capital ratio. As expected, the

[8]First of all, in 1961, they established two major banks, National Agricultural Cooperatives Federation and the Medium Industry Bank. The establishment led the rapid growth of bank credit. Second, they revised the Korea Development Bank's Charter resulting in massive inflow of capital from abroad. Finally and most importantly, the bank of Korea issued a guarantee to the foreign investor, which increased access of funds from private firms. See details Cole and Park (1983)

long run growth rate is lower and the equilibrium capital to human capital ratio increases as the countries are financially less developed because of over-accumulation of capital. In this case, self-financing diverts resources from innovation, causing aggregate inefficiency.

1.4 Quantitative Analysis

In this section, we will study the transitional dynamics of the model to evaluate the short-term effect and the long-term effect of financial reform. A large scale financial reform accelerates growth and leads to an innovation boom followed by an investment boom.

1.4.1 Time-varying equilibria

To consider transitional dynamics, we will introduce a time-dependent HJB equation where the interest rate can change over time.

Theorem 3. *(Time dependent HJB equation)*

$$\begin{aligned} \rho v_t(\tilde{a}) - \dot{v}_t(\tilde{a}) = \max_{c,k} log\, c &+ \frac{\partial v_t}{\partial \tilde{a}}(Ak^{1-\alpha} - r_t k + r_t \tilde{a} - c - x) \\ &+ \theta x (v_t(\frac{\tilde{a}}{\lambda}) - v_t(\tilde{a}) + \frac{1}{\rho} log(\lambda)) \\ &+ \psi_t(\tilde{a})(\mu a - k) \end{aligned} \tag{1.23}$$

Proof. See the Appendix □

The major difference between (1.20) and (1.44) is the presence of $\dot{v}_t(\tilde{a})$ which reflects the fact that the value function changes over time. Accordingly, the definition in Section 2.5 can be replaced by the following definition because the ratio $\tilde{a}$ is independent of the level of productivity z and innovation follows a random Poisson process.

Definition 1.4.1. *(Time-varying equilibria on transition)*
An equilibrium of this economy is an equilibrium path where the distribution of $\Omega_t(\tilde{a})$ and policy functions $\tilde{k}_t(\tilde{a}), \tilde{x}_t(\tilde{a}), \tilde{c}_t(\tilde{a})$ maximize the time dependent HJB equation (1.44) given the path of r_t, and satisfy:

Parameters	Values	Targets
ρ	0.01	Corresponding discount factor, $\beta = 0.99$
α	0.4	Bhandari and McGrattan (2018)
λ	15	Risk premium
A	0.011	Difference between pre-post reform BGP growth rate
θ	5.4	Friction-less BGP growth rate

Table 1.1: The summary of calibrated parameters

- *Capital market clearing condition*

$$\int \tilde{k}_t(\tilde{a})d\tilde{\Omega}_t(\tilde{a}) = \int \tilde{a} \ \ d\tilde{\Omega}_t(\tilde{a}) \tag{1.24}$$

- *Kolmogorov forward equation ($\tilde{\omega}_t$ is density of $\tilde{\Omega}_t$)*

$$\frac{\partial \omega_t(\tilde{a})}{\partial t} = \underbrace{-\frac{\partial}{\partial a}(\dot{\tilde{a}}_t(\tilde{a})\omega_t(\tilde{a})) - \theta\tilde{x}_t(\tilde{a})\omega_t(\tilde{a})}_{\textit{Outflow}} + \underbrace{\theta\tilde{x}_t(\lambda\tilde{a})\tilde{\omega}_t(\lambda\tilde{a})}_{\textit{Inflow}} \tag{1.25}$$

Following Achdou et al. (2017), we changed the algorithm to deal with the threshold problem and endogenous Poisson process. The detailed computational algorithm is presented in Appendix.B

1.4.2 Postreform Transitional dynamics

Calibration

In this section, we study the transitional dynamics of an unexpected financial reform following the same exercise conducted by Moll (2014). Before the reform, the economy stays at financial autarky ($\mu = 1$). The stark reform will be represented by a sudden increase in μ ($\geq \lambda$) to get the economy to the first best outcome on BGP. There are only 5 parameters to calibrate in the model. First of all, discount rate, ρ, is set to 0.01 so that it matches a 0.99 annual discount factor commonly used for the neoclassical growth model. The human capital share is a crucial parameter that governs the magnitude of Asian miracle the model

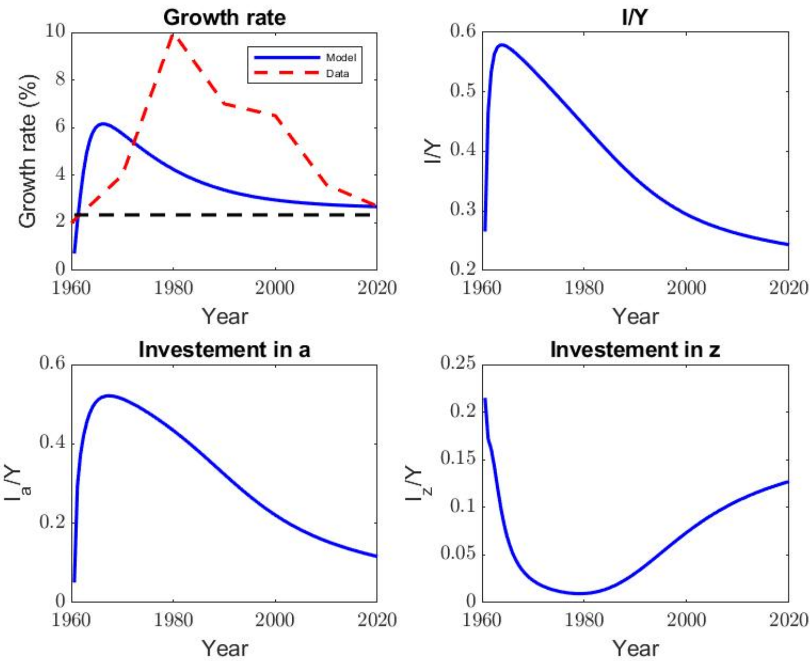


Figure 1.7: Transitional dynamics

is set up to predict. It is set to 0.4 following Bhandari and McGrattan (2018) where they estimate the ratio of intangible asset value to total asset value. The jump size variable, λ, is set to 15 to match 6 % risk premium between the 10 years US treasury yield and average S&P 500 return for 10 years. Finally, the total factor productivity, A, and innovation parameter, θ, are calibrated to match 0% growth before the reform and 2.5% BGP growth rate.

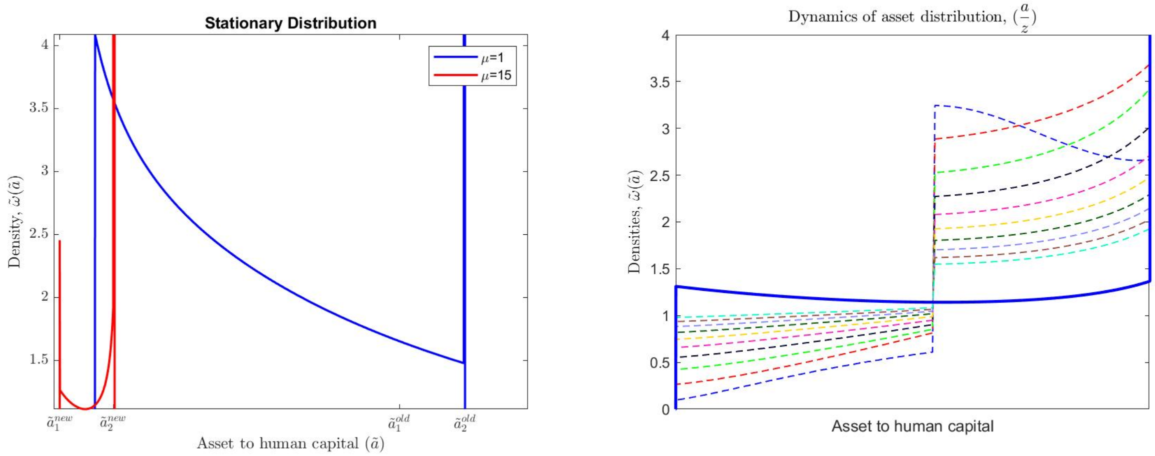
Result

Figure 1.7 shows the transitional dynamics. The results matches the growth experience of Asian countries well. Not surprisingly, the growth rate at date 0 is higher than the pre-reform growth rate because of immediate reallocation of capital. Next, an R&D boom follows the financial reform. This phenomenon can be explained by Figure 1.8a which shows the two stationary distribution of pre/post reform. Entrepreneurs who have too much capital compared to human capital start to recognize that capital does not have to be used as collateral. As we see, the agents above the new innovation threshold point will start to invest in innovation so that all the entrepreneurs above the point quickly move into the region of the post reform stationary distribution. This dynamic movement is well described in figure 1.8b. The R&D boom will accelerate the growth further by alleviating imbalance between aggregate capital and human capital, thereby increasing aggregate efficiency. It takes 30 years to converge to the new BGP growth rate, which is also consistent with the empirical evidence.

1.5 Conclusion

We develop an endogenous growth model to study the impact of financial constraints on economic growth through capital misallocation. To the best of our knowledge, this is the first paper encompassing the following features: 1) endogenous growth - related to long-run growth effect of the financial market imperfection, 2) persistent transition without any exogenous TFP assumption, 3) capturing the full picture of the Asian Miracle from pre-reform to the long term growth rates. Misallocation induced by capital market imperfection affects not only the level of output but also the growth rate of the economy at steady state. The model exhibits both transitory (short-term, but persistent) impacts as well as permanent impacts (long-term). Self-financing and overaccumulation of safe assets divert resources from optimal investment in R&D. This results in an imbalance between R&D and capital.

We also match the historical features of Asian countries' persistent rapid growth after financial reforms. There is a stark innovation boom following the reforms which leads the economy to balance its investment in capital and human capital. The aggregate inefficiency fades out as entrepreneurs start to invest in innovation and the growth rate eventually converges to a higher growth rate than the pre-reform growth rate.



(a) Stationary distributions

(b) Dynamics of asset distribution

Figure 1.8: Dynamics of distributions

1.6 Appendix A

1.6.1 Derivation of HJB

Individuals' decision can be represented with the following HJB equation

$$\begin{aligned}\rho V_t(a,z) - \dot{V}_t(a,z) = \max_{c,k,x} log\, c &+ \frac{\partial V_t}{\partial a}(Az^{\alpha}k^{1-\alpha} + r_t(a-k) - c - x) \\ &+ \theta\frac{x}{z}(V_t(\lambda z, a) - V_t(z,a)) \\ &+ \psi_t(\mu a - k)\end{aligned} \tag{1.26}$$

subject to

$$\psi_t(\mu a - k) = 0, \psi_t \geq 0 \tag{1.27}$$

Proof. Denote probability space $(\Omega, F,\ P)$ where Ω is a set of innovation outcomes starting at date t, F is a σ-field of Ω. $P : F \to [0,1]$ generated by Poisson process of (1.9) and (1.10).

Suppose $\{c_t^*(\omega), a_t^*(\omega), k_t^*(\omega), x_t^*(\omega)\}$ is the optimal solution of the problem where agents maximize his utility (1.4) subject to constraints (1.6) and (1.7), and Poisson jump process (1.9) and (1.10). Here ω is a sample path of Ω. Let us define the value function at time t as

$$V_t(a_t, z_t) = \int_\Omega \int_t^\infty e^{-\rho(s-t)} \ \ log \ \ c_s^*(\omega) \ \ dsdP(\omega) \tag{1.28}$$

Then

$$E_t[V_{t+\Delta}(a_{t+\Delta}^*(\omega), z_{t+\Delta}(\omega))] = \int_\Omega \int_{t+\Delta}^\infty e^{-\rho(s-(t+\Delta))} \ \ log \ \ c_s^*(\omega) \ \ dsdP(\omega) \tag{1.29}$$

Using the method of change variable, we can get

$$\begin{aligned} E_t[V_{t+\Delta}(a_{t+\Delta}^*(\omega), z_{t+\Delta}(\omega))] - V_t(a_t, z_t) = \int_\Omega \Big\{ & (1 - e^{-\rho\Delta}) \int_t^\infty e^{-\rho(s-t)} \ \ log \ \ c_{s+\Delta}^*(\omega) \ \ dsdP(\omega) \\ & - \int_t^{t+\Delta} e^{-\rho(s-t)} \ \ log \ \ c_s^*(\omega) \ \ ds \Big\} dP(\omega) \end{aligned} \tag{1.30}$$

After divide the equation by Δ and take a limit,

$$\lim_{\Delta \to 0} \frac{E_t[V_{t+\Delta}(a_{t+\Delta}^*(\omega), z_{t+\Delta}(\omega))] - V_t(a_t, z_t)}{\Delta} = \rho V_t(a_t, z_t) - \ \ log \ \ c_t^* \tag{1.31}$$

Please note that the probabilities of the number of jump received in a small time interval Δ at date t are $P(0;\Delta) = 1 - \theta \frac{x_t}{z_t} \Delta + o(\Delta)$, $P(1;\Delta) = \theta \frac{x_t}{z_t} \Delta + o(\Delta)$, and $\sum_j^\infty P(j:\Delta) = o(\Delta)$ where $o(\cdot)$ is a little o function. Using the definition of value function, equation (1.29) cab be expressed as

$$\begin{aligned} E_t[V_{t+\Delta}(a_{t+\Delta}^*(\omega), z_{t+\Delta}(\omega))] - V_t(a_t, z_t) = & (1 - \theta \frac{x_t}{z_t} \Delta + o(\Delta)) V_{t+\Delta}(a_{t+\Delta}^*(\omega), z_t) \\ & + (\theta \frac{x_t}{z_t} \Delta + o(\Delta)) V_{t+\Delta}(a_{t+\Delta}^*(\omega), \lambda z_t) + o(\Delta) \\ & - V_t(a_t, z_t). \end{aligned} \tag{1.32}$$

Rearranging and manipulating equation (1.32) lead

$$\begin{aligned}
E_t[V_{t+\Delta}(a^*_{t+\Delta}(\omega), z_{t+\Delta}(\omega))] - V_t(a_t, z_t) &= (1-\theta\frac{x^*_t}{z_t}\Delta + o(\Delta))\{V_{t+\Delta}(a^*_{t+\Delta}(\omega), z_t) - V_t(a^*_{t+\Delta}(\omega), z_t)\} \\
&+ (\theta\frac{x^*_t}{z_t}\Delta + o(\Delta))\{V_{t+\Delta}(a^*_{t+\Delta}(\omega), \lambda z_t) - V_t(a^*_{t+\Delta}(\omega), \lambda z_t)\} \\
&+ V_t(a^*_{t+\Delta}(\omega), z_t) - V_t(a_t, z_t) \\
&+ (\theta\frac{x^*_t}{z_t}\Delta + o(\Delta))\{V_t(a^*_{t+\Delta}(\omega), \lambda z_t) - V_t(a^*_{t+\Delta}(\omega), z_t)\} + o(\Delta)
\end{aligned} \tag{1.33}$$

After divide the equation by Δ and take a limit,

$$\lim_{\Delta\to 0}\frac{E_t[V_{t+\Delta}(a^*_{t+\Delta}(\omega), z_{t+\Delta}(\omega))] - V_t(a_t, z_t)}{\Delta} = \dot{V}_t(a_t, z_t) + \frac{\partial V_t}{\partial a}\dot{a}_t + \theta\frac{x^*_t}{z_t}(V_t(a^*_t(\omega), \lambda z_t) - V_t(a^*_t(\omega), z_t)) \tag{1.34}$$

Equation (1.31) and (1.34) imply

$$\rho V_t(a_t, z_t) - \dot{V}_t(a_t, z_t) = \ log\ c^*_t + \frac{\partial V_t}{\partial a}\dot{a}_t + \theta\frac{x^*_t}{z_t}(V_t(a^*_t(\omega), \lambda z_t) - V_t(a^*_t(\omega), z_t)) \tag{1.35}$$

where $\dot{a}_t$ is a budget constraint (1.10). Here we skip complementary-slackness condition (CSC) because variables are the optimal solution. However, when we define HJB recursively, we need to put CSC condition. $\square$

1.6.2 Proof of theorem 1

Theorem 1

A BGP of the economy is an equilibrium path where aggregate productivity level, $\bar{z}_t$, and aggregate capital $\bar{a}_t$ grow at a constant rate g and the interest rate r_t is constant

$$r = (1-\alpha)A(\frac{\bar{a}}{\bar{z}})^{-\alpha} \tag{1.36}$$

$$g = r - \rho \tag{1.37}$$

where $\frac{\bar{a}}{\bar{z}}$ is a solution for the following nonlinear equation

$$\underbrace{\theta(\lambda-1)\alpha A\left(\frac{\bar{a}}{\bar{z}}\right)^{1-\alpha} + \rho - \frac{\rho(\lambda-1)}{log\lambda}}_{\text{Risk-adjusted return from R\&D}} = \underbrace{(1-\alpha)A\left(\frac{\bar{a}}{\bar{z}}\right)^{-\alpha}}_{\text{Return from physical asset}} \tag{1.38}$$

Proof. This is a sketch of proof. Let us start to examine a special case where there are only two states, $(a, 0)$ and $(0, z)$. Given the equilibrium interest rate r,

$$\rho V(a,0) = \max_c logc + \frac{\partial V}{\partial a}(ra - c) \tag{1.39}$$

The well known analytic solution is

$$V(a,0) = \frac{1}{\rho}\log a + \frac{1}{\rho}\log\rho + \frac{r-\rho}{\rho^2}. \tag{1.40}$$

and the rate of capital accumulation is $\frac{\dot{a}}{a} = r - \rho$. On the other hand, entrepreneurs owns productivity

$$\rho V(0,z) = \max_x log(\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} r^{-\frac{1-\alpha}{\alpha}} z - x) + \theta\frac{x}{z}(V(0,\lambda z) - V(0,z))^{9} \tag{1.41}$$

[9]This is not generally true because entrepreneurs have an option to accumulate capital. It turns out that switching investment is not optimal in friction-less economy.

The corresponding analytic solution is

$$V(0,z)=\frac{1}{\rho}logz-\frac{1}{\rho}log(\frac{\theta}{\rho}log\lambda)+\frac{\theta}{\rho^2}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}A^{\frac{1}{\alpha}}r^{-\frac{1-\alpha}{\alpha}}log\lambda-\frac{1}{\rho}. \tag{1.42}$$

the rate of human capital accumulation is $E[\frac{\dot{z}}{z}]=\{\theta\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}A^{\frac{1}{\alpha}}r^{-\frac{1-\alpha}{\alpha}}-\frac{\rho}{\log\lambda}\}(\lambda-1)$. On the BGP, aggregate capital and human capital grow at the same rate. Therefore,

$$\{\theta\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}A^{\frac{1}{\alpha}}r^{-\frac{1-\alpha}{\alpha}}-\frac{\rho}{\log\lambda}\}(\lambda-1)=r-\rho \tag{1.43}$$

We can solve for r and get the optimal ratio of $\frac{a}{z}$ □

1.6.3 Proof of theorem 2 and 3

Theorem 3.

Individual behavior only depends on $\tilde{a}=\frac{a}{z}$ and $V_t(a,z)=v_t(\frac{a}{z})-\frac{1}{\rho}\ log\ z$ where $v_t(\tilde{a})$ follows

$$\begin{aligned}\rho v_t(\tilde{a})-\dot{v}_t(\tilde{a})=\max_{\tilde{c},\tilde{k},\tilde{x}} log\ \tilde{c}&+\frac{\partial v_t}{\partial\tilde{a}}(A\tilde{k}^{1-\alpha}-r_t\tilde{k}+r_t\tilde{a}-\tilde{c}-\tilde{x})\\&+\theta\tilde{x}(v_t(\frac{\tilde{a}}{\lambda})-v_t(\tilde{a})+\frac{1}{\rho}log(\lambda))\\&+\tilde{\psi}_t(\tilde{a})(\mu\tilde{a}-\tilde{k})\end{aligned} \tag{1.44}$$

Proof. Denote $\tilde{u}=\frac{u}{z}$ where u are variables. (For example, $\tilde{a}=\frac{a}{z}$.). Guess $V_t(a,z)=v_t(\frac{a}{z})+\frac{1}{\rho}logz$. Subtracting $logz$ from the both side of equation (1.26) yields

$$\begin{aligned}\rho V_t(a,z)-\dot{V}_t(a,z)-logz=\max_{c,k,x} log\ \tilde{c}&+\frac{\partial V_t}{\partial a}(Az^\alpha k^{1-\alpha}-r_tk+r_ta-c-x)\\&+\theta\frac{x}{z}(V_t(a,\lambda z)-V_t(a,z))\\&+\psi_t(a)(\mu a-k)\end{aligned} \tag{1.45}$$

Please note that $\frac{\partial V_t}{\partial a}=\frac{\partial v_t}{\partial a}\frac{1}{z}$ and $V_t(a,\lambda z)-V_t(a,z)=v_t(\frac{\tilde{a}}{\lambda})-v_t(\tilde{a})+\frac{1}{\rho}log(\lambda)$. Therefore,

$$
\begin{aligned}
\rho v_t(\tilde{a}) - \dot{v}_t(\tilde{a}) = \max_{\tilde{c},\tilde{k},\tilde{x}} log\ \tilde{c} &+ \frac{\partial v_t}{\partial \tilde{a}}(A\tilde{k}^{1-\alpha} - r_t\tilde{k} + r_t\tilde{a} - \tilde{c} - \tilde{x}) \\
&+ \theta\tilde{x}(v_t(\frac{\tilde{a}}{\lambda}) - v_t(\tilde{a}) + \frac{1}{\rho}log(\lambda)) \\
&+ \tilde{\psi}_t(\tilde{a})(\mu\tilde{a} - \tilde{k})
\end{aligned} \tag{1.46}
$$

□

1.6.4 Proof of corollary 2

Corollary 2.

When $r > \rho$, there exists $\tilde{a}_1^* \equiv \frac{1}{\mu}(\frac{(1-\alpha)A}{r})^{\frac{1}{\alpha}}$, $\tilde{a}_2^* \in \mathbb{R}^+$ and $\tilde{a}_2^* > \tilde{a}_1^*$

$$
\begin{aligned}
\rho v(\tilde{a}) = &\max_{\tilde{c}} log\ \tilde{c} + \frac{\partial v}{\partial \tilde{a}}(A\mu^{1-\alpha}\tilde{a}^{1-\alpha} - (\mu-1)r\tilde{a} - \tilde{c}) \text{ if } \tilde{a} \leq \tilde{a}_1^* \\
&\max_{c} log\ \tilde{c} + \frac{\partial v}{\partial \tilde{a}}(\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}A^{\frac{1}{\alpha}}r^{-\frac{1-\alpha}{\alpha}} + r\tilde{a} - \tilde{c}) \text{ if } \tilde{a}_1^* < \tilde{a} \leq \tilde{a}_2^* \\
&\max_{x} log\ (\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}A^{\frac{1}{\alpha}}r^{-\frac{1-\alpha}{\alpha}} + r\tilde{a} - \tilde{x}) + \theta\tilde{x}(v(\frac{\tilde{a}}{\lambda}) - v(\tilde{a}) + \frac{1}{\rho}log(\lambda)) \text{ if } \tilde{a}_2^* < \tilde{a}
\end{aligned} \tag{1.47}
$$

Proof. First of all, the financial constraint threshold is straightforward because the production choice is a purely static problem. The optimal amount of capital utilized for unconstrained firm should satisfy the first order condition,

$$
(1-\alpha)A\tilde{k}^{-\alpha} = r. \tag{1.48}
$$

Therefore, $k^* = (\frac{(1-\alpha)A}{r})^{\frac{1}{\alpha}}$. Therefore, entrepreneurs below than $\frac{k^*}{\mu}$ will borrow as much as they can (i.e constraints are binding).

Second, entrepreneurs invest all resources in either capital or innovation. This is simply because of the linearity of return from investment. The marginal return from capital is $\frac{\partial v}{\partial \tilde{a}}$ and the marginal return from innovation is $\theta(v(\frac{\tilde{a}}{\lambda}) - v(\tilde{a}) + \frac{1}{\rho}log(\lambda))$. Therefore, if $\frac{\partial v}{\partial \tilde{a}} >$

$\theta(v(\frac{\tilde{a}}{\lambda}) - v(\tilde{a}) + \frac{1}{\rho}log(\lambda))$. They will invest all resources in capital, and vice versa.

Third, we need to show that entreprenuers are accumulating capital when they have relatively low asset, $\tilde{a}$. Let us start to introduce the following lemma.

Lemma 1.

1) There exists ϵ s.t $\lim_{a\downarrow 0} v'(\tilde{a}) = \infty$ if $\tilde{a} \in [0, \epsilon)$

2) There exists ϵ s.t $v''(\tilde{a}) < 0$ if $\tilde{a} \in [0, \epsilon)$

3) $\lim_{a\downarrow 0} \theta(v(\frac{\tilde{a}}{\lambda}) - v(\tilde{a}) + \frac{1}{\rho}log(\lambda)) = \frac{\theta}{\rho}log(\lambda)$

4) $\theta(v(\frac{\tilde{a}}{\lambda}) - v(\tilde{a}) + \frac{1}{\rho}log(\lambda)) < \frac{\theta}{\rho}log(\lambda)$

proof of lemma 1

1) The optimal condition for consumption is $\frac{1}{\tilde{c}^*} = max(v'(\tilde{a}), \theta(v(\frac{\tilde{a}}{\lambda}) - v(\tilde{a}) + \frac{1}{\rho}log(\lambda))$. Because $v(0) = 0$ and $c^* = 0$ as $\tilde{a} \to 0$, $\lim_{\tilde{a}\downarrow 0} v'(\tilde{a}) = \infty$

2) The second property comes from the 1st property of lemma 1 and $v''(\tilde{a}) = -\frac{1}{c^*(\tilde{a})}\frac{\partial c^*(\tilde{a})}{\partial \tilde{a}}$.

3) This is straight forward because v(0)=0

4) $v(\cdot)$ is increasing.

The first two properties imply that marginal return from capital is bigger than marginal return from innovation when $\tilde{a}$ is low.

Finally, we want to show $\tilde{a}_1^* < \tilde{a}_2^*$. Suppose this is not true. Then, we can find the smallest $\tilde{a}_2^* > 0$ satisfying

$$\rho v(\tilde{a}) = \max_x log\,(A\mu^{1-\alpha}\tilde{a}^{1-\alpha} - (\mu-1)r\tilde{a} - \tilde{x}) + \theta\tilde{x}(v(\frac{\tilde{a}}{\lambda}) - v(\tilde{a}) + \frac{1}{\rho}log(\lambda)) \tag{1.49}$$

Please note that agent, $\tilde{a} \in [0, a_2^*)$ invests in capital because of 1) of Lemma 1. Equation (1.49) is the case where they are financially constrained, but investing in innovation. Differentiating equation (1.49) with respect $\tilde{a}$ implies

$$\rho v'(\tilde{a}_2^*) = \frac{1}{\tilde{c}^*}\Big\{(1-\alpha)A\mu^{1-\alpha}\tilde{a}_2^{*-\alpha} - (\mu-1)\Big\} + \theta\tilde{x}^*(\frac{1}{\lambda}v'(\frac{\tilde{a}_2^*}{\lambda}) - v'(\tilde{a}_2^*)). \tag{1.50}$$

Because $v''(\cdot) < 0$ and $v'''(\cdot) > 0$ if $\tilde{a} \in [0, \epsilon)$, the last term, $\theta x^*(\frac{1}{\lambda}v'(\frac{\tilde{a}_2^*}{\lambda}) - v'(\tilde{a}_2^*))$, is positive. Also, the marginal return from asset, $(1-\alpha)A\mu^{1-\alpha}\tilde{a}^{-\alpha} - (\mu - 1) > r$ because firms are financially constrained. This implies

$$\rho v'(\tilde{a}_2^*) > \frac{r}{\tilde{c}^*} = r\theta(v(\frac{\tilde{a}_2^*}{\lambda}) - v(\tilde{a}_2^*) + \frac{1}{\rho}log\lambda). \tag{1.51}$$

Rearranging

$$\frac{v'(\tilde{a})}{\theta(v(\frac{\tilde{a}}{\lambda}) - v(\tilde{a}) + \frac{1}{\rho}log\lambda)} > \frac{r}{\rho} > 1 \tag{1.52}$$

which implies return from capital is higher than return from innovation. This is a contradiction to the assumption that there is a financially constrained firm who invests in innovation.

□

1.6.5 Proof of proposition 1

Proposition 1.

If $\mu \geq \lambda$, $g = g^{\text{optimal}}$ where g^{optimal} is the friction-less aggregate growth rate.

Proof.

Let us introduce Lemma 2.

Lemma 2.

1) $\tilde{\omega}(\tilde{a}) > 0$ for any $\tilde{a} \in U \equiv [\frac{\tilde{a}_2^*}{\lambda}, \tilde{a}_2^*]$ and $\tilde{\omega}(\tilde{a}) = 0$ for $\tilde{a} \notin U$

2)Given the equilibrium interest rate r, the innovation threshold point $\tilde{a}_2^*$ is greater than $\mu\tilde{a}_1^*$

proof of lemma 2

1) From corollary 2, $\tilde{x}^*(\tilde{a}_2^*) > 0$ and $\tilde{x}(\tilde{a}) = 0$ if $\tilde{a} \neq \tilde{a}_2^*$. Moreover, $\dot{\tilde{a}} > 0$ for any $\tilde{a} \in (0, \infty)$. Intuitively, this implies the budget constraint has always positive drift (outflow) and the only inflow is coming from $\tilde{a}_2^*$. Therefore, there is no mass outside $[\frac{\tilde{a}_2^*}{\lambda}, \tilde{a}_2^*]$

2) If $\tilde{a}_2^* < \mu\tilde{a}_1^*$, KMF equation (1.22) implies $\tilde{\omega}(\tilde{a}) = 0$ for $\tilde{a} \in [\mu\tilde{a}_1^*, \infty)$ from 1) of Lemma 2.

Because $\tilde{k}^*(\tilde{a}) > \tilde{a}$ for $\tilde{a} \in [0, \mu\tilde{a}_1^*)$, $\int \tilde{k}^*(\tilde{a})d\tilde{\Omega}(\tilde{a}) > \int \tilde{a} \ d\tilde{\Omega}(\tilde{a})$ which contradicts the market clearing condition.

Assume $g < g^{optimal}$ and $\mu \geq \lambda$. $g < g^{optimal}$ implies that there is strictly positive mass in the constrained region, $\exists\ \tilde{a}_f \in (0, \tilde{a}_1^*)$ where $\tilde{\omega}(\tilde{a}_f) > 0$. Otherwise, the constrained problem coincides with the unconstrained problem which is the case of friction-less economy, therefore, the collateral constraint doesn't play any role. On the other hand, because of 2) of Lemma 2

$$\frac{\tilde{a}_2^*}{\lambda} \geq \frac{\tilde{a}_2^*}{\mu} > \frac{\mu\tilde{a}_1^*}{\mu} = \tilde{a}_1^*$$

Therefore, $\tilde{\omega}(\tilde{a}) = 0$ if $\tilde{a} \leq \tilde{a}_1^*$. This is a contradiction to $\tilde{\omega}(\tilde{a}_f) > 0$.

$\square$

1.7 Appendix B

1.7.1 Stationary HJB equation

We extend a finite difference method based on Achdou et al. (2017). Specifically, we take the implicit method for computational efficiency and it doesn't rely on the updating size for value function iteration, Δ. The problem, (1.47) is different from theirs in two ways : 1) Poisson intensity is endogenous 2) The boundary, $\tilde{a}_2^*$ is endogenous. Given the boundary point, $\tilde{a}_2^{j*}$ where where j is the index for the possible boundary points, $j = 1, ..., J$. We approximate (1.47) with I discrete points. Therefore, the grid points lie between a_{min} and $\tilde{a}_2^{j*}$. Starting from $v^0 = (v_1^0, ..., v_I^0)$, we update the value function following

$$\frac{v_i^{(j),n+1} - v_i^{(j),n}}{\Delta} + \rho v_i^{(j),n+1} = log\ c_i^{(j),n} + \frac{v_{i+1}^{(j),n+1} - v_i^{(j),n+1}}{\Delta a} \underbrace{(A\mu^{1-\alpha}\tilde{a}_i^{1-\alpha} - (\mu - 1)r\tilde{a}_i - c_i^{(j),n})}_{s_i} \text{ if } \tilde{a}_i \leq \tilde{a}_1^*$$

$$= log\ c_i^{(j),n} + \frac{v_{i+1}^{(j),n+1} - v_i^{(j),n+1}}{\Delta a} \underbrace{(\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} r^{-\frac{1-\alpha}{\alpha}} + r\tilde{a}_i - c_i^{(j),n})}_{s_i} \text{ if } \tilde{a}_1^* \leq \tilde{a}_i \leq \tilde{a}_2^*$$

$$= log\ c_I^{(j),n} + \theta \underbrace{(\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} r^{-\frac{1-\alpha}{\alpha}} + ra_I - c_I^{(j),n})}_{x} (v(\frac{a_I}{\lambda}) - v_I^{(j),n} + \frac{1}{\rho} log(\lambda)) \text{ if } a_i = \tilde{a}_2^{j*}$$

Δ is the step size for update and Δa is the distance between grid points. Please note that $a_I = \tilde{a}_2^j$. The optimal consumption $c_i^{(j),n}$, can be computed by $(\frac{v_{i+1}^{(j),n} - v_i^{(j),n}}{\Delta a})^{-1}$ We can rewrite down the problem, (1.47), as the following matrix form.

$$\frac{1}{\Delta}(v^{n+1} - v^n) + \rho v^{n+1} = u^n + H^n v^{n+1}$$

where

$$u^n = \begin{bmatrix} log\ c_1^n \\ \vdots \\ log\ c_I^n + \theta x \frac{1}{\rho} log(\lambda) \end{bmatrix}, H^n = \begin{bmatrix} -\frac{s_1}{\Delta a} & \frac{s_1}{\Delta a} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & -\frac{s_2}{\Delta a} & \frac{s_2}{\Delta a} & \cdots & \cdots & \cdots & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & \cdots & \cdots & \cdots & -\frac{s_{I-1}}{\Delta a} & \frac{s_{I-1}}{\Delta a} \\ 0 & 0 & 0 & \theta x & \cdots & \cdots & -\theta x \end{bmatrix}$$

which can be rewritten as

$$B^n v^{n+1} = b^n, B^n = (\frac{1}{\Delta} + \rho)I - H^n, b^n = u^n + \frac{1}{\Delta} v^n.$$

We update the value function until $v^{(j),n+1}$ is arbitrary close to $v^{(j),n+1}$. Finally, we repeat this exercise for J possible endogenous points and choose the correct boundary point, $a_2^{j'}$ where it maximizes v_i at every grid points.

1.7.2 Kolmogorov Forward Equation

We are looking for a stationary distribution. The corresponding KMF equation is

$$0 = \underbrace{-\frac{\partial}{\partial a}(\dot{\tilde{a}}(\tilde{a})\omega(\tilde{a})) - \theta x(\tilde{a})\omega(\tilde{a})}_{\text{Outflow}} + \underbrace{\theta x(\lambda\tilde{a})\tilde{\omega}(\lambda\tilde{a})}_{\text{Inflow}} \tag{1.53}$$

We discretize this as

$$0 = -[s_i g_i]' + \theta x g_I 1(i = i') - \theta x g_I 1(i = I) \text{ for } i = 2, ..., I-1 \tag{1.54}$$

where $[s_i g_i]'$ is approximated as

$$\frac{g_i s_i - g_{i-1} s_{i-1}}{\Delta a} \tag{1.55}$$

i' is the indicator index for $\frac{a_2}{\lambda}$ representing the closest grid point to $\frac{\tilde{a}_2^*}{\lambda}$. The system of equation from (1.53) to (1.55) can be easily computed through $H^T \mathrm{g} = 0$ where H^T is the transpose of the transition matrix we get from Appendix B.1. Finally, we can obtain the stationary distribution by calculating the eigenvector of H^T and satisfying $g'\Delta a = 1$ which represents the mass should sum up to 1.

1.7.3 Stationary equilibrium

We use the well known algorithm following Huggett (1993) and Aiyagari (1994). Given the interest rate r, we can get the stationary HJB equation and KMF equation and use them to compute the stationary distribution, g. In equilibrium, the capital market should be cleared which means

$$\underbrace{\sum_i^I k_i^*(a_i) g_i \Delta a}_{AggregateDemand} = \underbrace{\sum_i^I a_i g_i \Delta a}_{AggregateSupply} \tag{1.56}$$

Therefore, if aggregate demand is greater than the waaggregate supply, we update interest rate by increasing, and vice versa until it converges.

CHAPTER 2

Micro to Macro: Do network linkages matter?

Aggregate fluctuations are a major feature of the economy, and a large portion of macroeconomic research has been devoted to analyzing the origin of these fluctuations. While economy-wide shocks (e.g. monetary policy, oil shock and war) have commonly been thought to be an important driver to induce aggregate fluctuations, individual shocks have been overlooked because they would be averaged out among many agents. Several novel papers such as Johnson (2014)and Durlauf (1993) try to generate aggregate fluctuations from the micro-level, but these models have not been widely accepted by economists due to the strong assumptions of the models, their tractability or their inability to match the data.

However, two pioneering recent papers, Gabaix (2011) and Acemoglu et al. (2012), have offered a challenge to this dominant point of view. Gabaix (2011) focuses on the implications of Zipf's law distribution of firm size, first noticed by Axtell (2001). Based on his random growth theory (Gabaix (1999)), which provides a theoretical base for Zipf's law, he argues that when the distribution of firms has fat tails, the dissipating rate of idiosyncratic shocks follows $\ln N$ instead of $\sqrt{N}$, which implies a much slower convergence rate. Subsequently, he generates a measure of firm level, idiosyncratic shocks, which he calls the granular residual, in order to inspect the co-movement between these shocks and aggregate fluctuations.

On the other hand, Acemoglu et al. (2012) focus on the asymmetric network structure of firms in the economy. LikeGabaix (2011), the dissipating rate of idiosyncratic shocks might become much slower as the network structure grows unbalanced. Upstream firms productivity change might induce aggregate shocks even if the number of firms goes to infinity (Cascading effect).

To give some intuition on the difference between these two ideas, Gabaix (2011) is concerned with the fact that GM is a big company, while Acemoglu et al. (2012) focuses on its influential linkages to other companies (which may or may not be related to its size). This paper begins from the premise that each of these factors plays an important role. Ignoring either piece would result in an incomplete picture of the impact of firms on the economy. The goal of the paper is to develop a tractable multi-sector model that can explore the effects of firm size and sectoral linkages simultaneously.

Even though monopolistic competition is widely used in the heterogeneous firms literature, it has not been frequently employed in the granularity literature. Gaubert and Itskhoki (2016) incorporate the feature in the finite firm level framework, but they don't have network linkages between firms or sectors and their focus is more on comparative advantage rather than idiosyncratic shocks. Di Giovanni et al. (2014) also emphasize the importance of the network with empirical evidence under monopolistic competition.

Most closely related to this paper is Baqaee (2014), which shares a similar setup, but there are two key differences. First, he focuses on the extensive margin of firm entry and exit, while this paper confines analysis to existing firms in the spirit of Gabaix (2011). The setup of the model in this paper is also much more conducive to empirical analysis, which is absent in Baqaee (2014). Finally, this paper directly corresponds to the recent literature.

One might think size already includes the information of the input-output structure because intermediate demand is also another determinant of firm sales as well as consumer demand and productivity. In the competitive market, the argument is true - sales of a firm is a sufficient statistic for the influence of an individual company,which was proved by Hulten (1978) in the competitive market. However, this paper shows that in the presence of monopolistic competition, Hulten's theorem doesn't hold. Therefore, network linkages matter when it comes to granularity.

In section 2, I'll set up the basic framework of a multi-sector monopolistic competition model. Section 3 will directly clarify the linkage between the model and the existing literature through variance decomposition. Finally, I will define the new granular residual driven by the model, and then use it to conduct empirical analysis and compare it with Gabaix's previous measure.

2.1 Model

I extend Acemoglu et al. (2012) by introducing a multi-sector version of Melitz (2003). For simplicity, I assume there is no entry and exit. One might also think the model as a static version of Long and Plosser (1983) incorporating monopolistically competitive heterogeneous firms.

2.1.1 Household

The representative household inelastically supplies 1 unit of labor and has Cobb-Douglas preferences over S sector goods.

$$u(C) = A \prod_{i=1}^{S} C_i^{\beta_i} \tag{2.1}$$

C_i is a consumption aggregator originating from sector i. A is a constant, which will be defined later to normalize price index. The agent maximizes his utility subject to the following budget constraint.

$$\sum_{i=1}^{S} P_i C_i = W + \Pi$$

P_i is the price index of consumption aggregator i. Wage is denoted by W. Π is aggregate profit from monopolistic firms in this economy.

$$\Pi = \sum_{i=1}^{S} \int_0^1 \pi_{u,i} du$$

2.1.2 CES aggregator in sector s

There is a continum of varieties in each sector. To analyze granularity, the assumption of infinitely many firms will be modified in section 3. The composite good follows the form of

Dixit and Stiglitz (1977).

$$Q_i = [\int_0^1 q_{u,i}^{\frac{\eta-1}{\eta}} du]^{\frac{\eta}{\eta-1}} \tag{2.2}$$

$q_{u,i}$ is the quantity of goods firm u produces in sector i. η is the elasticity of substitution between two varieties . Given the set of price of each variety, $p_{u,i}$, agents want to maximize Q_i across varieties. As a result, the price index of specific sector product is

$$P_i = [\int_0^1 p_{u,i}^{1-\eta} du]^{\frac{1}{1-\eta}} \tag{2.3}$$

Finally, demand for an individual variety, $q_{u,i}$ is

$$q_{u,i} = (\frac{p_{u,i}}{P_i})^{-\eta} Q_i \tag{2.4}$$

The economy can use each good either to consume or use as an intermediate input to produce other goods.

2.1.3 Firms

The technology of each firm follows constant returns to scale. However, they need to use labor and CES aggregated sector goods as intermediate inputs to produce. Each variety in sector i is produced by the monopolistically competitive firm, u. That is, the technology of individual firm u in sector i has the following technology.

$$q_{u,i} = z_{u,i} l_{u,i}^{\alpha} \prod_{j=1}^{S} X_{u,i,j}^{\omega_{ij}(1-\alpha)} \tag{2.5}$$

$X_{u,i,j}$ is the intermediate input from sector j that a firm u in sector i demands in order to produce its own good. $\omega_{i,j}$ is the share of good j, which sum to 1 to maintain constant return to scale. Each monopolistically competitive firm wants to maximize his profit given demand for his variety. Unlike Acemoglu et al (2012), heterogenity arises through the level of technology $z_{u,i}$ and unlike Gabaix(2011), there are network linkages across sectors through the intermediate goods. The profit maximization problem is given by

$$\pi_{u,i} = \max_{p_{u,i}} \; p_{u,i}q_{u,i} - Wl_{u,i} - \sum_{j=1}^{S} P_j X_{u,i,j} \tag{2.6}$$

subject to the production technology equation (5) and the demand of his variety (4).

Firm's optimal solution in sector i

$$p_{u,i} = \frac{\eta}{\eta - 1}\frac{MC_i}{z_{u,i}} \tag{2.7}$$

$$MC_i = (\frac{W}{\alpha})^{\alpha}(\frac{\xi_i}{1-\alpha})^{1-\alpha} \tag{2.8}$$

$$\xi_i = \prod_{j=1}^{S}(\frac{P_j}{\omega_{ij}})^{\omega_{ij}} \tag{2.9}$$

$$q_{u,i} = (\frac{p_{u,i}}{P_i})^{-\eta}Q_i \tag{2.10}$$

$$l_{u,i} = (\frac{\alpha}{1-\alpha})^{1-\alpha}(\frac{\xi_i}{W})^{1-\alpha}\frac{q_{u,i}}{z_{u,i}} \tag{2.11}$$

$$X_{u,i,j} = \frac{\xi_i\omega_{ij}}{P_j}(\frac{1-\alpha}{\alpha})^{\alpha}(\frac{W}{\xi_i})^{\alpha}\frac{q_{u,i}}{z_{u,i}} \tag{2.12}$$

Although it appears complicated, this optimal solution can be summarized by three intuitive ideas: (1) price is a constant markup over marginal cost. (2) quantity is determined by the firm's relative pricing position over aggregate price. (3) $\frac{Wl_{u,i}}{P_jX_{u,i,j}} = \frac{\alpha}{(1-\alpha)\omega_{ij}}$ or in words, wage payment to intermediate expenditure ratio depends only on the ratio of labor and the intermediate share parameter and doesn't differ across firms endowed with different technology in the same sector. The proof is almost similar to other monopolistic competition literature except multiple inputs.

2.1.4 General Equilibrium

The general equilibrium of the economy consists of

(1) A set of prices

The price index of CES aggregated goods $\{P_i\}_{i=1}^{i=S}$

The wage W

(2) A set of allocations

Consumption bundle $\{c_i\}_{i=1}^{i=S}$

Labor $\{\{l_{u,i}\}_u\}_{i=1}^{i=S}$

Intermediate Goods $\{\{X_{u,i,j}\}_u\}_{j=1}^{j=S}\}_{i=1}^{i=S}$

such that

1) The representative household maximizes his utility

2) Individual monopolistically competitive firms in each sector maximize profits

3) Markets clear

Labor:

$$\sum_{i=1}^{s}\int_0^1 l_{u,i}du = 1$$

Goods in each sector:

$$C_i + \sum_{j=1}^{S}\int_0^1 X_{u,j,i}du = Q_i \quad \text{for} \quad \forall i = 1, \cdots, S$$

4) Aggregator consistency

$$Q_i = \big[\int_0^1 q_{u,i}^{\frac{\eta-1}{\eta}} du\big]^{\frac{\eta}{\eta-1}}$$

$$P_i = \big[\int_0^1 p_{u,i}^{1-\eta} du\big]^{\frac{1}{1-\eta}}$$

Analysis of General Equilibrium

In the setup of Acemoglu et al. (2012), they showed the logarithm of real value added is the sum of log sectoral shocks adjusted by it's influence. I show that even in the environment of monopolistic competition, the same implication arises.

The proof here follows a similar method. The result looks similar, but is different in that

Acemoglu et al. (2012) don't have the feature of the firm heterogenity and monopolistic competition.

Proposition 1 (Logarithm of real value added)

$$y = \log(C/P) = v'\epsilon \tag{2.13}$$

where

$$v' = \alpha\beta'[I - (1-\alpha)\Omega]^{-1}, \;\; \Omega_{(i,j)} = \omega_{ij}, \;\; \epsilon_i = \frac{1}{\alpha} log\Big(\Big\{\int_0^1 z_{u,i}^{\eta-1} du\Big\}^{\frac{1}{\eta-1}}\Big)$$

Proof: See the appendix

One interesting result of both Acemoglu et al. (2012) and Gabaix (2011) is that its influence vector is equal to the sales ratio. This is the result of Hulten's theorem (1978). Therefore, firm sales (size) to GDP ratio is sufficient statistic to calculate the contribution of individual firm on economy. In other words, size already includes the information of network channels.

However, Hulten(1978)'s theorem doesn't hold in the presence of monopolistic competition. The following proposition is one of the primary contributions of this paper.

Proposition 2

In the presence of monopolistic competition, firm size is not sufficient statistic to measure the individual firm's impact on economy. That is, there is discrepancy between influence vector and sales vector.

$$v' = \alpha\beta'[I - (1-\alpha)\Omega]^{-1}$$

$$s' = \frac{1+\alpha(\eta-1)}{\eta}\beta'[I - (1-\alpha)\frac{\eta-1}{\eta}\Omega]^{-1}$$

Proof: See the appendix

s' is the equilibrium sales vector as ratio of GDP. We can easily see that influence vector and sales fraction becomes equalized when economy becomes more competitive as a limiting case. (i.e η goes to infinity). The proposition 3 tells us that firm size relative to whole economy under/over-estimate the impact of each sector. To understand actual influence of each firm, we need details of input-output network between sector. That is, the influence structure is important as much as large firms are important.

2.2 Linkage with recent literature of granularity

To analyze granularity, we need to change the continuous firm assumption and allow for a finite number of firms. In this case, there can be a small difference in the optimal solution of the firm. However, to simplify our discussion, here I impose an *ad-hoc* pricing rule where mark up is constant. This assumption ensures that all propositions hold even in the finite firm level case. To get a more accurate result, we should use the variable mark-up approach.[1] However, this approach is beyond the scope of this paper so that I leave this adjustment for future research.

As we have shown in (13), the log of GDP is the linear combination of the sectoral influence component and the geometric mean of individual firm technology in each sector. Suppose firm growth follows the following process

$$\log(z_{k,i,t}) - \log(z_{k,i,t-1}) \equiv g_{k,i,t} = G_t + g_{i,t} + e_{k,i,t} \tag{2.14}$$

where k is the individual firm index, G_t is the macro shock which influences firm commonly across sector and individual firm, the sectoral shock is denoted by g_{it}, and $e_{u,i,t}$ is the idiosyncratic shock that will eventually form the granular residual , which is what we are interested in. To measure the pure effect of an individual shock, I impose the further assumption that

[1]Still, constant mark-up can be a good approximation of variable mark-up. As the number of firms in a sector increases, it's error term decreases. For the literature on finite firm level pricing, Atkeson and Burnstein (2008) and Gaubert and Itskhoki (2016) are good references. The former uses variable mark-up with oligopolistic competition in quantities, the latter uses same approach, but with competition in pricing. I'll include this feature in future research.

the shocks are independent. Finally, I assume that $e_{k,i,t}$ has mean 0 and finite variance, which follows the spirit of Gibrat's law used by Gabaix (1999) to exhibit the Zipf's law of firm size distribution (Random growth theory). For notational convenience, I assume z^{α} is the technology level of the firm follows instead of z. This helps to cancel out $\frac{1}{\alpha}$ in front of ϵ, but the underlying logic would remain as same as before. Equivalently we can assume

$$\log(z_{k,i,t}) - \log(z_{k,i,t-1}) \equiv g_{k,i,t} = \alpha(G_t + g_{i,t} + e_{k,i,t})$$

Proposition 3 (Decomposition of variance of growth)

Suppose individual firm growth follows the process (14). Further, G_t, g_{it}, $e_{k,i,t}$ are independent from each other. $e_{k,i,t}$ is also independent across firms. Then,

$$\sigma^2_{\Delta(log(y_t))} = \sigma^2_G + \sum_{i=1}^{S} v_i^2 \sigma^2_{g,i} + \sum_{i=1}^{S}\sum_{k=1}^{N_i} v_i^2 (\frac{S_{k,i,t-1}}{Y_i})^2 \sigma^2_{e,k,i} \tag{2.15}$$

where

$S_{k,i,t-1}$: Sales of k firm in sector i at t-1

$Y_{i,t-1}$: Aggregate real value added of sector i at t-1

N_i : Number of firm in sector i

$v' = \alpha\beta'[I - (1-\alpha)\Omega]^{-1}$: Influence vector

$\sigma^2_{\Delta(log(y_t))}$: Variance of growth in GDP.

σ^2_G : Variance of macro shock.

$\sigma^2_{g,i}$: Variance of sectoral shock.

$\sigma^2_{e,k,i}$: Variance of k firm's idiosyncratic shock in sector i.

Proof: See the appendix

This proposition tells us that aggregate fluctuation can be explained by 1) Aggregate shocks 2) Sectoral shocks and 3) Idiosyncratic shocks which is the 'new' granular residual slightly different from other literature. The last term is the channel through which an individual shock can affect aggregate fluctuation.

$$\sum_{i=1}^{S}\sum_{k=1}^{N_i} v_i^2 (\frac{S_{k,i,t-1}}{Y_{i,t}})^2 \sigma_{e,k,i}^2 \tag{2.16}$$

Like the other literature, it depends on firm size, $S_{i,k}$, but unlike the other literature, the shock can be propagated through network linkages. It is noteworthy that the influence vector, v' becomes s'(sector sales to GDP ratio) when the economy becomes perfect competition (i.e η goes to infinite). Then, the last term becomes

$$\sum_{k=1}^{N} (\frac{S_{kt}}{GDP_t})^2 \sigma_{e,k}^2$$

where N becomes total number of firm existing in the economy and i is the index of the firm in any sector. We can easily see sales to GDP ratio becomes a sufficient statistic to measure the impact of idiosyncratic shocks on the economy in this case. Therefore, the granular residual of Gabaix (2011) and Acemoglu et al. (2012) is the limiting case of the 'new' granular residual produced by the model.

2.3 Empirical Analysis

The main goal of this section is to compare the 'New' granular residual driven by the model and the existing granular residual from Gabaix (2011). To do so, I use the same data set as the paper, but a different data cleaning method, which gives a more precise representation.

2.3.1 Empirical implementation

To calculate labor productivity of firm k in i sector, I use a naive proxy for the labor productivity most literature used.

$$z_{k,i,t} = \frac{\text{sales of firm k in year t}}{\text{Number of workers in firm k in year t}}$$

Therefore, we can calculate growth rate of productivity by taking log difference denoted by $\gamma_{k,i,t} \equiv (\log(z_{k,i,t}) - \log(z_{k,i,t-1}))$ and $\gamma_{k,i,t}$ follows the growth process (14).

Even though we have full specification of variance decomposition (15), we cannot conduct a full variance decomposition analysis. The following argument will clarify this point of view. According to the setup, $\hat{\gamma}_{i,t} \equiv \frac{1}{N_i}\sum_{k=1}^{N_i}\gamma_{k,i,t}$ is a consistent estimator of $G_t + g_{it}$. On the other hand, one might measure G_t with $\hat{G}_t \equiv \frac{1}{N}\sum_{i=1}^{S}\sum_{k=1}^{N_i}\gamma_{k,i,t}$ where $N = \sum_{i=1}^{S} N_i$. However, it is not a consistent estimator of G_t with respect to N_i. To get a consistent estimator, we need the additional assumption that the number of sectors also goes to infinity or that there are no sector shocks. That is, aggregate effects are not identifiable even though the number of firms is infinite. (i.e $\hat{G}_t \xrightarrow{p} G_t + q'g_t$). g_t is the S×1 consisting of sector shock at time t. q' is the speed of divergence of N_i.[2] Therefore, I want to focus on contribution of idiosyncratic shocks to GDP, which is fully identifiable and our main interest of this paper.

For comparison with Gabaix (2011), I consider the case where there is no sectoral shock (i.e $g_{it} = 0$). Then, the new granular residual becomes

$$\Gamma_t^{New} = \sum_{i=1}^{S}\sum_{k=1}^{N_i} v_i\left(\frac{S_{k,i,t-1}}{Y_{i,t}}\right)(\gamma_{k,i,t} - \hat{G}_t) \tag{2.17}$$

This is equal to Gabaix(2011)'s measure when the influence vector is equal to sales vector.

$$\Gamma_t^{Gabaix} = \sum_{k=1}^{N}\left(\frac{S_{k,t-1}}{Y_t}\right)(\gamma_{k,t} - \hat{G}_t) \tag{2.18}$$

On the other hand, the new granular residual with sector demeaning corresponding to (16) is

$$\Gamma_t^{New} = \sum_{i=1}^{S}\sum_{k=1}^{N_i} v_i\left(\frac{S_{k,i,t-1}}{Y_{i,t}}\right)(\gamma_{k,i,t} - \hat{\gamma}_{i,t}) \tag{2.19}$$

[2] Intuitively, you can think q'g_t is the weighted average sector shock depending on number of firms in each sector.

$$\Gamma_t^{Gabaix} = \sum_{i=1}^{S}\sum_{k=1}^{N_i}(\frac{S_{k,t-1}}{Y_t})(\gamma_{k,t} - \hat{\gamma}_{k,i,t}) \tag{2.20}$$

2.3.2 Data & Result

To calculate (17) - (20), we need to calculate the influence vector, $v' = \alpha\beta'[I - (1-\alpha)\Omega]^{-1}$. Ω is the input-output matrix which is available from the I-O accounts data of the Bureau of Economic Analysis (BEA). Following Acemoglu et al. (2012), I use commodity-by-industry direct requirements from 1997 to 2014. [3]. Also, I use the use table valued at purchasers prices to get the personal expenditure data by sector. Finally, I set α equal to 0.66 which is the labor share of GDP.

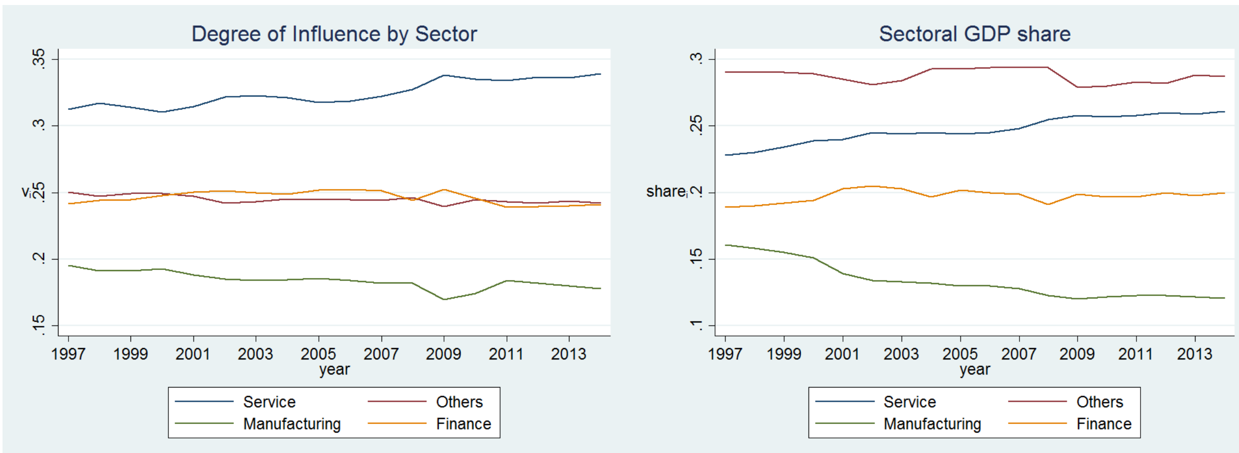
Figure 1 shows discrepancy between sales and the influence vector driven by network linkages, which is contrast to existing literature. For example, GDP share underestimates the impact of the service sector, manufacturing and finance sector. However, it overestimates the impact of the other sectors.[4] In this experiment, the mean of the influence vector from 1997 to 2007 will be used for v_i, because detail data is not available before 1997.

For firm level productivity, I use the annual U.S. Compustat data from 1951 to 2014. I drop the firms that do not have employment and sales data for the previous year. I use the price index and real GDP from BEA. Then, I calculate real sales revenue of each firm, taking 2009 price level as a base. For comparison with Gabaix (2011), I choose top 100 firms with respect to net sales in the last period, replacing any drops with the next highest firm, and measure individual firm's productivity to calculate granular residual. However, I use the mean productivity growth of all firms for $\hat{G}_t$ or γ_{it}, which is an efficient estimator. As Gabaix mentions in his paper, for the case of extreme extraordinary events such as merging among companies, I winsorize the productivity growth rates at 40%.

---

[3]"The direct requirements indicates the amount of a commodity that is required to produce a dollar of the industry's output", BEA

[4]The other sectors include agriculture, mining, utilities, transport, whole-sale trade, retail trade, information and construction.



(a) Network influence, v_i (b) Sectoral GDP ratio, s_i

Figure 2.1: Network influence and Sectoral GDP ratio

	(1)	(2)	(3)	(4)
(intercept)	0.029**	0.033**	0.029**	0.029**
	(0.0024)	(0.0023)	(0.0025)	(0.0025)
Γ_t^{New}	1.887**	2.230**	0.940	1.006*
	(0.6728)	(0.5971)	(0.4798)	(0.4867)
Γ_{t-1}^{New}	2.296**	1.716**	1.500**	1.629**
	(0.6730)	(0.5941)	(0.4787)	(0.4981)
Γ_{t-2}^{New}		1.780**		0.249
		(0.5978)		(0.4841)
N	63	62	63	62
R^2	0.2436	0.3435	0.1632	0.1793
$Adj.R^2$	0.2184	0.3096	0.1354	0.1542

Table 2.1: Explanatory power of new granular residual on GDP growth

	(1)	(2)	(3)	(4)
(intercept)	0.029**	0.033**	0.029**	0.029**
	(0.0024)	(0.0025)	(0.0024)	(0.0026)
Γ_t^{Gabaix}	1.995**	2.021**	0.890	1.008*
	(0.6736)	(0.5502)	(0.4938)	(0.5001)
Γ_{t-1}^{Gabaix}	2.115**	1.292*	1.423*	1.640**
	(0.6736)	(0.5524)	(0.4867)	(0.5106)
Γ_{t-2}^{Gabaix}		1.561*		0.4075
		(0.5523)		(0.4891)
N	63	62	63	62
R^2	0.2213	0.2859	0.1425	0.1698
$Adj.R^2$	0.1953	0.2490	0.1140	0.1268

Table 2.2: Explanatory power of Gabaix's granular residual on GDP growth

As Gabaix (2011) mentioned, if aggregate fluctuation were solely driven by economy-wide shocks, the granular residual should be uncorrelated with growth rate of GDP. Table 1 shows the results from the regression of GDP growth on the new granular residual. The first two specifications are the cases where there is no sector shock. The last two specifications are about the granular residual with industry demeaning. Table 2 shows the result regression of GDP growth on the granular residual devised by Gabaix (2011).[5] As we see, the correlation between growth and the measure of idiosyncratic shocks is high, which means that the performance of individual firms, especially large firms, matters.

Overall, the new granular residual explains better than the previous granular residual. This is the result of the effect of network linkage. The new granular residual without industry shock explains 34 % of GDP. However, in the presence of sectoral shock, it's explanatory power drops down to 18 %, which implies sectoral shock also matters, and sectoral shock and network-linkages is another interesting topic to think about.

[5]This result is different from his paper because I calculate average firm productivity from whole population rather than Top 100 or 1000 firm.

2.4 Concluding Remarks

This paper develops a monopolistic competition model where there are network linkages through intermediate inputs. In the presence of monopolistic competition, information of sales is not sufficient information of individual firm's impact on economy. Network linkages are as important as the size of the firm. The measured discrepancy between influence and sales ratio supports this argument.

The new analysis sheds light on precision of granularity literature. With the measure driven by model, this paper re-examined the importance of idiosyncratic shock in large firm. As a result, the modified granular residual explains almost 35 % of aggregate fluctuations without presence of sectoral shock. However, it decreases to 18 % once these shocks are considered. Even though it still remains as meaningful correlation, the drop indicates the importance of sectoral shocks, which might lead us to another interesting topic.

Other extensions could include employing variable mark-up, refining data work and doing a full variance decomposition analysis to increase precision. Moreover, dynamic variant with entry and exit might improve our understanding of aggregate fluctuations.

2.5 Appendix

Lemma 1.

$$\frac{W}{\Pi} = \alpha(\eta - 1)$$

Proof of lemma 1:

Individual firm's profit from firm's optimal solution

$$\pi_{u,i} = \frac{1}{\eta^{\eta}(\eta-1)^{1-\eta}} z_{u,i}^{\eta-1} \Big(\frac{MC_i}{P_i}\Big)^{1-\eta} P_i Q_i$$

Therefore, aggregate profit in sector i is simply

$$\pi_i = \frac{1}{\eta^\eta(\eta-1)^{1-\eta}}(\frac{MC_i}{P_i})^{1-\eta}P_iQ_i\Big\{\int_0^1 z_{u,i}^{\eta-1}du\Big\}$$

Finally, aggregate profit in this economy becomes

$$\Pi = \frac{1}{\eta^\eta(\eta-1)^{1-\eta}}\sum_{i=1}^{S}(\frac{MC_i}{P_i})^{1-\eta}P_iQ_i\Big\{\int_0^1 z_{u,i}^{\eta-1}du\Big\}$$

On the other hand,

$$l_{u,i} = (\frac{\alpha}{1-\alpha})^{1-\alpha}(\frac{\xi_i}{W})^{1-\alpha}\frac{q_{u,i}}{z_{u,i}} = (\frac{\alpha}{1-\alpha})^{1-\alpha}(\frac{\xi_i}{W})^{1-\alpha}(\frac{MC_i}{P_i})^{-\eta}(\frac{\eta}{\eta-1})^{-\eta}z_{u,i}^{\eta-1}Q_i$$

$$= \alpha(\frac{\eta}{\eta-1})^{-\eta}W^{-1}(\frac{MC_i}{P_i})^{1-\eta}z_{u,i}^{\eta-1}P_iQ_i$$

If we aggregate across varieties and sectors, and further use labor market clearing condition.

$$W = \alpha(\frac{\eta}{\eta-1})^{-\eta}\sum_{i=1}^{S}(\frac{MC_i}{P_i})^{1-\eta}P_iQ_i\Big\{\int_0^1 z_{u,i}^{\eta-1}du\Big\}$$

Proposition 1 (Logarithm of real value added)

$$y = \log(C/P) = v'\epsilon$$

where

$$v' = \alpha\beta'[I-(1-\alpha)\Omega]^{-1},\ \ \Omega_{(i,j)} = \omega_{ij},\ \ \epsilon_i = \frac{1}{\alpha}log\Big(\Big\{\int_0^1 z_{u,i}^{\eta-1}du\Big\}^{\frac{1}{\eta-1}}\Big)$$

Proof:

Optimal labor equation (11) yields

$$l_{u,i} = (\frac{\alpha}{1-\alpha})^{1-\alpha}(\frac{\xi_i}{W})^{1-\alpha}\frac{q_{u,i}}{z_{u,i}} = (\frac{\alpha}{1-\alpha})^{1-\alpha}(\frac{\xi_i}{W})^{1-\alpha}z_{u,i}^{\eta-1}(\frac{MC_i}{P_i})^{-\eta}(\frac{\eta}{\eta-1})^{-\eta}Q_i$$

To simplify, define aggregate intermediate goods which firm u used in sector i

$$I_{u,i} \equiv \prod_{j=1}^{S} X_{u,i,j}^{\omega_{ij}} = (\frac{1-\alpha}{\alpha})^{\alpha}(\frac{W}{\xi_i})^{\alpha}\frac{q_{u,i}}{z_{u,i}} = (\frac{1-\alpha}{\alpha})^{\alpha}(\frac{W}{\xi_i})^{\alpha}z_{u,i}^{\eta-1}(\frac{MC_i}{P_i})^{-\eta}(\frac{\eta}{\eta-1})^{-\eta}Q_i$$

Plug optimal labor and optimal aggregate intermediate goods he chooses into his production function.

$$q_{u,i} = z_i l_{u,i}^{\alpha} I_{u,i}^{1-\alpha} = z_{u,i}^{\eta}(\frac{MC_i}{P_i})^{-\eta}(\frac{\eta}{\eta-1})^{-\eta}Q_i$$

With CES aggregator defined in (2), we can get

$$Q_i = \{\int_0^1 q_{u,i}^{\frac{\eta-1}{\eta}}du\}^{\frac{\eta}{\eta-1}} = \Big\{\int_0^1 z_{u,i}^{\eta-1}du\Big\}^{\frac{\eta}{\eta-1}}(\frac{MC_i}{P_i})^{-\eta}(\frac{\eta}{\eta-1})^{-\eta}Q_i \qquad (2.21)$$

With definition of marginal cost MC_i and equation (21) , we can get wage equation in each sector i

$$W^{\alpha} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\frac{\eta-1}{\eta}P_i\xi_i^{-(1-\alpha)}\Big\{\int_0^1 z_{u,i}^{\eta-1}du\Big\}^{\frac{1}{\eta-1}}$$

Take logs on both sides

$$\alpha\log(W) = \alpha\log(\alpha) + (1-\alpha)\log(1-\alpha) + \log(\frac{\eta-1}{\eta}) + \log(P_i) + \\ (1-\alpha)\sum_{j=1}^{S}\omega_{ij}\log(\omega_{ij}) - (1-\alpha)\sum_{j=1}^{S}\omega_{ij}\log(P_j) + \log(\Big\{\int_0^1 z_{u,i}^{\eta-1}du\Big\}^{\frac{1}{\eta-1}}) \tag{2.22}$$

By lemma 1,

$$\alpha\log(W+\Pi) = \alpha\log(\frac{1+\alpha(\eta-1)}{\alpha(\eta-1)}W) = \alpha\log(1+\alpha(\eta-1)) - \alpha\log(\alpha(\eta-1)) + \alpha\log(W)$$

Finally, we can get combined household income equation. For any industry i,

$$\alpha\log(W+\Pi) = B + \log(P_i) + (1-\alpha)\sum_{j=1}^{S}\omega_{ij}\log(\omega_{ij}) - (1-\alpha)\sum_{j=1}^{S}\omega_{ij}\log(P_j) + \log(\Big\{\int_0^1 z_{u,i}^{\eta-1}du\Big\}^{\frac{1}{\eta-1}}) \tag{2.23}$$

where $B = (1-\alpha)\log(1-\alpha) + \alpha\log(1+\alpha(\eta-1)) + (1-\alpha)\log(\eta-1) - \log(\eta)$

Finally, we can get similar form of Acemoglu et al. (2012). The remainder of the proof is exactly same as theirs (see Acemoglu et al. (2012)).

Define influence vector $v' = \alpha\beta'[I-(1-\alpha)\Omega]^{-1}$ where I is S $\times$ S identity matrix. Ω is S $\times$ S input requirements matrix. After multiplying each equation by i th element of influence vector, if we sum over all sectors, it gives

$$\log(W+\Pi) = v'\epsilon + \mu$$

where $\mu = \sum_{i=1}^{S}\beta_i\log(P_i) + B/\alpha + \frac{1-\alpha}{\alpha}\sum_{i=1}^{S}\sum_{j=1}^{S}v_i\omega_{ij}\log(\omega_{ij})$ and ϵ is defined above
Normalize price index $(\prod_{i=1}^{S}\beta_i^{-\beta_i}A^{-1})\prod_{i=1}^{S}P_i^{\beta_i} = 1$ (i.e $P_i^{\beta_i} = A\prod_{i=1}^{S}\beta_i^{\beta_i}$).
This makes $\mu = 0$. Finally, we prove that the logarithm of real value added is

$$\log(W+\Pi) = v'\epsilon$$

Proposition 2.

In the presence of monopolistic competition, firm size is not a sufficient statistic to measure the individual firm's impact on economy. That is, there is a discrepancy between influence vector and sales vector.

$$v' = \alpha\beta'[I - (1-\alpha)\Omega]^{-1}$$

$$s' = \frac{1+\alpha(\eta-1)}{\eta}\beta'[I - (1-\alpha)\frac{\eta-1}{\eta}\Omega]^{-1}$$

proof)

In each sector, goods market clearing condition tells us

$$P_iC_i + \sum_{j=1}^{S}\int_0^1 P_iX_{u,j,i}du = P_iQ_i \quad \text{for} \quad \forall i = 1, \cdots, S$$

The representative agent spends β portion of his income to buy i sector goods and an individual firm spends $(1-\alpha)\omega_{ji}$ portion of his marginal cost to buy intermediaries.

$$\beta_i(W+\Pi) + \sum_{j=1}^{S}\int_0^1 (1-\alpha)\omega_{ji}\frac{MC_j}{z_{u,j}}q_{u,j}du = P_iQ_i$$

By equation (7) and (10),

$$\beta_i(W+\Pi) + \sum_{j=1}^{S}\int_0^1 (1-\alpha)\omega_{ji}\frac{\eta-1}{\eta}p_{u,j}^{1-\eta}P_j^{\eta}Q_jdu = P_iQ_i$$

$$\beta_i(W+\Pi) + (1-\alpha)\frac{\eta-1}{\eta}\sum_{j=1}^{S}\omega_{ji}P_jQ_jdu = P_iQ_i$$

Define $f_i = \frac{P_iQ_i}{W+\Pi}$,

$$\beta + (1-\alpha)\frac{\eta-1}{\eta}\Omega' f = f$$

$$f' = \beta'[I - (1-\alpha)\frac{\eta-1}{\eta}\Omega]^{-1}$$

Normalize f' so that it's sum equal to 1. (i.e $s' = \frac{1+\alpha(\eta-1)}{\eta} f'$)

Proposition 3 (Decomposition of variance of growth)

Suppose individual firm growth follows the process (14). Further, G_t, g_{it}, $e_{k,i,t}$ are independent each other. $e_{k,i,t}$ is also independent across firms. Then,

$$\sigma^2_{\Delta(log(y_t))} = \sigma^2_G + \sum_{i=1}^{S} v_i^2 \sigma^2_{g,i} + \sum_{i=1}^{S} \sum_{k=1}^{N_i} v_i^2 (\frac{S_{k,i,t-1}}{Y_i})^2 \sigma^2_{e,k,i} \tag{2.24}$$

where

$S_{k,i,t-1}$: Sales of k firm in sector i at t-1

$Y_{i,t-1}$: Aggregate real value added of sector i at t-1

N_i : Number of firm in sector i

$v' = \alpha\beta'[I - (1-\alpha)\Omega]^{-1}$: Influence vector

$\sigma^2_{\Delta(log(y_t))}$: Variance of growth in GDP.

σ^2_G : Variance of macro shock.

$\sigma^2_{g,i}$: Variance of sectoral shock.

$\sigma^2_{e,k,i}$: Variance of k firm's idiosyncratic shock in sector i.

Proof:

At the finite firm level, the equation (13) becomes

$$\Delta \log(y_t) = \sum_{i=1}^{S} v_i \log\Big(\{\sum_{k=1}^{N_i} z^{\eta-1}_{k,i,t} / \sum_{k'=1}^{N_i} z^{\eta-1}_{k',i,t-1}\}^{\frac{1}{\eta-1}}\Big) = \sum_{i=1}^{S} v_i \frac{1}{\eta-1} \log\Big(\sum_{k=1}^{N_i} \frac{z^{\eta-1}_{k,i,t-1}}{\sum_{k'=1}^{N_i} z^{\eta-1}_{k',i,t-1}} \Big(\frac{z_{k,i,t}}{z_{k,i,t-1}}\Big)^{\eta-1}\Big)$$

Note that $\frac{z^{\eta-1}_{k,i,t-1}}{\sum\limits_{k'=1}^{N_i} z^{\eta-1}_{k',i,t-1}} = \frac{S_{k,i,t-1}}{Y_{i,t-1}}$, $\sum\limits_{k=1}^{N_i} S_{k,i,t-1} = Y_{i,t-1}$, and $\sum\limits_{i=1}^{S} v_i = 1$.

Therefore,

$$\Delta \log(y_t) = \sum_{i=1}^{S} v_i \frac{1}{\eta-1} \log\Big(\sum_{k=1}^{N_i} \frac{S_{i,k,t-1}}{Y_i} \Big(\frac{z_{k,i,t}}{z_{k,i,t-1}}\Big)^{\eta-1}\Big) = \sum_{i=1}^{S} v_i \frac{1}{\eta-1} \log\Big(\sum_{k=1}^{N_i} \frac{S_{i,k,t-1}}{Y_i} \Big(\frac{z_{k,i,t}}{z_{k,i,t-1}}\Big)^{\eta-1}\Big)$$

Imposing the shock process (14) gives us

$$\Delta \log(y_t) = G_t + \sum_{i=1}^{S} v_i g_{i,t} + \sum_{i=1}^{S} v_i \frac{1}{\eta - 1} \log\Big(\sum_{k=1}^{N_i} \frac{S_{i,k,t-1}}{Y_i} exp^{e_{k,i}(\eta-1)}\Big)\Big)$$

Take the first Taylor approximation with respect to $e_{k,i}$

$$\Delta \log(y_t) = G_t + \sum_{i=1}^{S} v_i g_{i,t} + \sum_{i=1}^{S} v_i \Big(\sum_{k=1}^{N_i} \frac{S_{i,k,t-1}}{Y_i} e_{k,i}\Big) + o(e)$$

$$Var(\Delta \log(y_t)) = \sigma_G^2 + \sum_{i=1}^{S} v_i^2 \sigma_{g,i}^2 + \sum_{i=1}^{S}\sum_{k=1}^{N_i} v_i^2 (\frac{S_{i,k,t-1}}{Y_i})^2 \sigma_{e,k,i}^2$$

CHAPTER 3

The Role of Sectoral Linkages in International Trade

3.1 Introduction

From Ricardo's basic comparative advantage model to more recent rich quantitative analysis, the trade literature has almost always centered around a common theme: 'Trade improves our lives.' However, although a variety of different methods have been proposed, there is little consensus among trade economists regarding the source and the size of the gains from trade.

In one of the most comprehensive explorations of the welfare gains from trade, Arkolakis et al. (2012) inspect gains from trade quantitatively in a wide class of recent trade models. They demonstrate that welfare gain can be measured by two statistics: the Domestic share (λ), and the trade elasticity (ϵ). $\hat{\mathbf{W}} = \lambda^{-1/\epsilon}$. Following ACR, Costinot and Rodriguez-Clare (2013) empirically measure the welfare gain with various models and market structures. Without fully specifying the general equilibrium, they provide a method to measure welfare gain. Quantitatively, estimated welfare gain is small overall, but adding multiple sectors and tradable intermediate goods allows them to get decent welfare gains from trade.

The role of intermediate goods has also been emphasized by many other papers. In the macro literature, for example, Acemoglu et al. (2012) analyze asymmetric network structure and its implications for idiosyncratic shocks. Di Giovanni et al. (2014) apply a similar idea to French firm level date to give a quantitative assessment of the importance of firm linkages on aggregate fluctuations. The results from each of these papers suggest that sectoral linkages can have economically important implications.

In the international trade literature, earlier work in this area has been done by Balistreri et al. (2011), who estimate gains from trade with intermediate goods, but without sectoral linkages. More closely related to this paper, Ossa (2015)incorporates intermediate goods and sectoral linkages into an Armington model, emphasizing the importance of varying trade elasticities across sectors. In a similar analysis, Caliendo and Parro (2012) analyze NAFTA's tariff reductions using a Ricardian sectoral linkages model. Both of these papers demonstrate that the connections between different industries can increase the gains from trade relative to standard formulations.

Based on this literature, I develop a multi-sector model that builds on Melitz (2003) by adding input-output linkages across sectors. More specifically, I examine the interaction between tradable goods and non-tradable goods through network linkages. In the model, these linkages set off a positive chain reaction that amplifies gains from trade.

The model in this paper is different from existing models with input-output linkages in that it can not only compare welfare gains with standard models, but also enables counterfactual analysis to examine the importance of network linkages. My results show that the intermediate goods channel alone cannot generate realistic welfare gains and therefore understanding the role of sectoral linakages appears to be necessary to fully understand the dynamics of international trade. Another interesting finding is that higher network dependence on tradable goods can create substantially larger welfare gains. However, dependence on tradable goods from non-tradable sector is low and a large portion of consumption expenditure (80 % in the U.S) comes from non-tradable goods, which limits the ability of my model to fully capture the potential gains from trade.

To summarize, the model is the first to include all of the following features simultaneously:

1. Multi-sector model with firm heterogenity in the spirit of Melitz (2003)

2. Input-output linkages

3. Intermediate goods and consumption goods that are both tradable

4. Fully specified general equilibrium analysis

Combining all of these features provides a closer look at the impact of sectoral linkages on welfare gain and non-tradable goods, which I believe has not been fully exploited by the literature to this point.

In section 2, I set up the model incorporating the features mentioned above. The benchmark calibration and computational solution procedure will be explained in section 3. Finally, various counter-factuals and analysis will be conducted in section 4.

3.2 Framework

This paper extends a multi-sector version of Melitz (2003) with network linkages. The network structure follows Long and Plosser (1983) and Acemoglu et al. (2012). To simplify the discussion, there are only 2 countries. The foreign version of each variable will be denoted by a * above the corresponding variable. Although I do not solve the model with more than two countries, similar logic should apply.

3.2.1 The model

2.1.1. Households. The representative household has Cobb-Douglas preference over S sector goods and inelastically supplies L units of labor.

$$U = \sum_{i=1}^{S} \beta_i log(C_i) \tag{3.1}$$

β_i is the preference parameter on consumption goods from sector i. The agent maximizes his utility subject to the following budget constraint

$$\sum_{i=1}^{S} P_i C_i = WL + \Pi \tag{3.2}$$

where P_i is the price index of consumption aggregator i and W is wage. Π is aggregate profit from domestic monopolistic firms, given by

$$\Pi = \sum_{i=1}^{S} \int_{u \in M_i} \pi_i(u)\, du \tag{3.3}$$

where M_i indicates the set of domestic firms in sector i.

2.1.2. Composite goods. In each sector, there is a continuum of varieties provided by monopolistically competitive firms. As in Acemoglu et al. (2012), the composite goods can either be consumed or used as intermediates to produce a specific variety final good. Each good follows the standard Dixit and Stiglitz aggregator.

$$Q_i = [\int_{u \in \Psi_i} q_i(u)^{\frac{\eta-1}{\eta}}\, du]^{\frac{\eta}{\eta-1}} \tag{3.4}$$

where u is an index for each firm. $q_i(u)$ is the quantity of the variety produced by firm, u, η is the elasticity of substitution between two varieties (same across country and sectors[1]), and Ψ_i is the set of varieties domestic consumer can access in their home market. Subsequently, the counterpart of this set in a foreign country will be denoted by Ψ_i^*.

In the standard model, total expenditure is entirely derived from consumer spending, which means it is equivalent to real value added. However, in the presence of intermediate inputs, total expenditure corresponds to gross output, which can be decomposed into consumption and intermediate demand. Given the home gross expenditure of sector i, E_i, the home expenditure on a variety is given by

$$p_i(u)q_i(u) = (\frac{p_i(u)}{P_i})^{1-\eta} E_i \tag{3.5}$$

where $p_i(u)$ is the price imposed by an individual firm, and the sectoral aggregate price index is

$$P_i = \Big(\int_{u \in \Psi_i} p_i(u)^{1-\eta} du\Big)^{\frac{1}{1-\eta}} \tag{3.6}$$

[1]One might can generalize by allowing substitution parameters to be different across sectors, but it would affect the Leontief matrix. This will be taken into account in future research

2.1.3 Firms   Following the benchmark model of Gaubert and Itskhoki (2016), I assume there is an exogeneous mass of firms M_i. Each firm in sector i draws productivity, φ from a Pareto distribution with a lower bound, $\underline{\varphi}_i$, discussed by Chaney (2008) Therefore, the cumulative distribution of productivity in sector i is given by

$$G_i(\varphi) = 1 - \big(\frac{\varphi}{\underline{\varphi}_i}\big)^{-\theta} \tag{3.7}$$

Also, the technology level of sector i can be summarized by

$$T_i = M_i \underline{\varphi}_i^{\theta} \tag{3.8}$$

where θ is the shape parameter of the firm distribution.

Firms are price setters, given demand equation (5). They use labor and intermediate goods to produce a variety of the final good. Importantly, they need intermediate inputs not only from their own sector, but also from other sectors, causing input-output linkages. Therefore, a firm has the following constant return to scale technology.

$$q_i(u) = \varphi_i(u) l_i(u)^{\alpha} \prod_{j=1}^{S} x_{ji}(u)^{\omega_{ji}(1-\alpha)} \tag{3.9}$$

where l_i is the amount of labor in firm u, $x_{ji}(u)$ is the demand of intermediary j from industry i, α is the labor share parameter, and ω_{ji} shapes the share of sector j good necessary to produce the efficient unit of sector i good.

In order to operate, each firm is required to pay F units of home labor. Additionally, to sell their product in the foreign markets, they need to pay F^* units of labor to foreign labor (so the payment goes to foreign workers). Finally, to deliver one unit of their good, they need to deliver τ_i units of a good from the domestic port. Therefore, the cost function of domestic firm endowed with φ in sector i can be summarized by

$$\iota_i(q) = \frac{mc_i}{\varphi} q + WF, \qquad \iota_i^{ex}(q) = \frac{mc_i \tau_i}{\varphi} q + W^* F^* \tag{3.10}$$

where

$$mc_i = (\frac{W}{\alpha})^{\alpha}(\frac{\xi_i}{1-\alpha})^{1-\alpha} \tag{3.11}$$

$$\xi_i = \prod_{j=1}^{S}(\frac{P_j}{\omega_{ji}})^{\omega_{ji}} \tag{3.12}$$

ι_i is the cost to produce q units of domestic goods and ι_i^{ex} is the cost to produce q units of foreign goods. Each cost function is active only when a firms operates or when they become an exporter, respectively. As we can see, mc_i is the channel through which one sectors actions can affect other sectors. The standard Melitz (2003) corresponds to the case when α is equal to 1 (No intermediate inputs needed in production). With the summarized version of the cost function, the firm's optimal price to maximize his profit is

$$p_i(u) = \frac{\eta}{\eta-1}\frac{mc_i}{\varphi_i(u)} \tag{3.13}$$

And the optimized profit will be

$$\begin{aligned} \pi_i(u) = &\Big[(\frac{\eta}{\eta-1})^{1-\eta}(\frac{mc_i}{P_i})^{1-\eta}\varphi_i(u)^{\eta-1}E_i - WF\Big]^+ \\ &+ \Big[(\frac{\eta}{\eta-1})^{1-\eta}(\frac{mc_i}{P_i})^{1-\eta}\varphi_i(u)^{\eta-1}E_i^* - W^*F^*\Big]^+ \end{aligned} \tag{3.14}$$

Finally, the input payment follows

$$Wl_i(u) = \alpha\varphi_i(u)^{\eta-1}(\frac{\eta}{\eta-1})^{-\eta}\Big(\frac{mc_i}{P_i}\Big)^{1-\eta}E_i \tag{3.15}$$

$$P_j x_{ji}(u) = (1-\alpha)\omega_{ji}\varphi_i(u)^{\eta-1}(\frac{\eta}{\eta-1})^{-\eta}\Big(\frac{mc_i}{P_i}\Big)^{1-\eta}E_i \tag{3.16}$$

3.2.2 Sectoral Equilibrium

Given the price level, equation (14) determines the cut-off level of the firm in each country. The sectoral equilibrium has a similar form to standard setups. (see detailed proof in

Appendix)

$$P_i = \frac{\eta}{\eta-1} mc_i \Big(\frac{\theta T_i}{\theta-\eta+1}\Big)^{-\frac{1}{\theta}} (1-\Phi_i)^{\frac{1}{\theta}} \Big(\frac{\eta W F}{E_i}\Big)^{\frac{\theta-\eta+1}{\theta(\eta-1)}} \tag{3.17}$$

where

$$\Phi_i = \frac{1}{1 + \big(\tau_i mc_i^*/mc_i\big)^{\theta} T_i/T_i^*} \tag{3.18}$$

Φ_i is the foreign share of domestic market.

The interpretation of sectoral price equations is almost the same as usual. High marginal cost, mc_i increases overall price index of the composite good. A higher technology level decreases the price, but this is partially canceled out by the domestic share term $1 - \Phi_i$. The last term $\left(\frac{\eta W F}{E_i}\right)$ becomes constant in the standard literature. Here, it depends on the network linkages, which will be discussed later.

3.2.3 General Equilibrium

To get the general equilibrium conditions, it's helpful to understand how the firm's revenue can be decomposed. With the aggregation of (14) - (16) and cutoff level derived in the Appendix, we can decompose revenue into several parts. Table 1 shows sector level of firm's accounting table

Therefore, the associated sector labor demand equation is

$$WL_i = \big(\alpha\frac{\eta-1}{\eta} + \frac{\theta-\eta+1}{\eta\theta}\big)[1-\Phi_i]E_i + \frac{\theta-\eta+1}{\eta\theta}\Phi_i E_i + \alpha\frac{\eta-1}{\eta}\Phi_i^* E_i^* \tag{3.19}$$

The first term is labor income, which includes both variable cost and fixed cost for domestic firms. The second term is the fixed cost payment from foreign firms and the final term is the wage paid to exporting goods.

The trade balance should satisfy

$$\sum_{i=1}^{S} NX_i = 0 \tag{3.20}$$

	Revenue from home market & foreign market $\big[1-\Phi_i\big]E_i+\Phi_i^*E_i^*$
Wage payment	$\alpha(\frac{\eta-1}{\eta})\ \big(\big[1-\Phi_i\big]E_i+\Phi_i^*E_i^*\big)$
Intermediate payment	$(1-\alpha)(\frac{\eta-1}{\eta})\ \big(\big[1-\Phi_i\big]E_i+\Phi_i^*E_i^*\big)$
Entry cost (Home)	$\frac{\theta-\eta+1}{\theta\eta}(1-\Phi_i)E_i$
Entry cost (Foreign)	$\frac{\theta-\eta+1}{\theta\eta}\Phi_i^*E_i^*$
Profit	$\frac{\eta-1}{\theta\eta}\big(\big[1-\Phi_i\big]E_i+\Phi_i^*E_i^*\big)$

Table 3.1: Domestic Firm's Revenue Split

where $NX_i=\Phi_i^*E_i^*-\Phi_iE_i$.

Unlike standard models, the expenditure ratio is not equal to a fixed portion of income. The sectoral expenditure equation follows

$$E_i=\beta_i(WL+\Pi)+\sum_{j=1}^{S}P_iX_{ij} \tag{3.21}$$

The left hand side indicates gross expenditure in sector i. On the right side, the first term represents consumer's spending on good i and the last term indicates the aggregate expenditure of sector j on intermediate good i. With equation (20) and (21), sector expenditure follows

$$E_i=v_iE+\sum_{j=1}^{S}\Gamma_{ij}NX_j \tag{3.22}$$

$\nu\equiv\frac{1+\alpha(\eta-1)}{\eta}[I-(1-\alpha)\frac{\eta-1}{\eta}\Omega]^{-1}\beta$ and $\Gamma\equiv[I-(1-\alpha)\frac{\eta-1}{\eta}\Omega]^{-1}-I$ where Ω is the input-ouput matrix whose (i,j) element is ω_{ij}.

ν is the inverse of the Leontief matrix used to calculate the input-output network, which is similar to Acemoglu et al. (2012) (Note that its elements sum to 1). Γ captures the added effect on network linkages with other sectors that arises when firms begin to export. Under the positive net exports of other sectors, the sector's gross expenditure becomes higher as linkages become stronger.[2]

To completely characterize the general equilibrium, a specific form for (3.22) would be required (see Appendix A).

$$e = \nu + \Big\{\tilde{\Phi}^* + \Gamma^{-1} + \tilde{\Phi}\Big\}^{-1}\Big\{\tilde{\Phi}^*\frac{W^*L^*}{WL} - \tilde{\Phi}\Big\}\nu \tag{3.23}$$

$\tilde{\Phi}$ is a foreign share matrix whose ith diagonal element is Φ_i and every other element 0.

e is an S$\times$1 vector of sector expenditure to gross expenditure ratios. (i.e $E_i = e_i E$). Clearly, as α goes to 1, it collapses into the standard case, where expenditure is a fixed fraction β_i of total expenditure.

Finally, the sectoral labor supply (3.19) and the trade balance equation (3.20) yields the labor market clearing condition

$$WL = \Big(\alpha\frac{\eta - 1}{\eta} + \frac{\theta - \eta + 1}{\eta\theta}\Big)E \tag{3.24}$$

which means wage payment is the fixed fraction of aggregate gross output.

3.3 Data and Calibration

The empirical portion of this paper considers the case of the US vs the rest of the world. I measure and calibrate several parameters to match the foreign share as a benchmark case, and then do counter-factual analysis to see the network effect on trade.

I use 2015 World Input-Output Data (WIOD) to measure intermediate share ω_{ij} for each sector and country. WIOD includes data for 40 countries and 34 sectors [3]. To simplify

---

[2] The simple example would be the case when there is only input-output linkage within sector. In this case, The equation (3.22) becomes $E_i = v_i E + (\frac{\eta}{1+\alpha(\eta-1)} - 1)NX_i$, which means positive net export sector has higher gross expenditure share.

[3] Public Admin and Defence; Compulsory Social Security is excluded

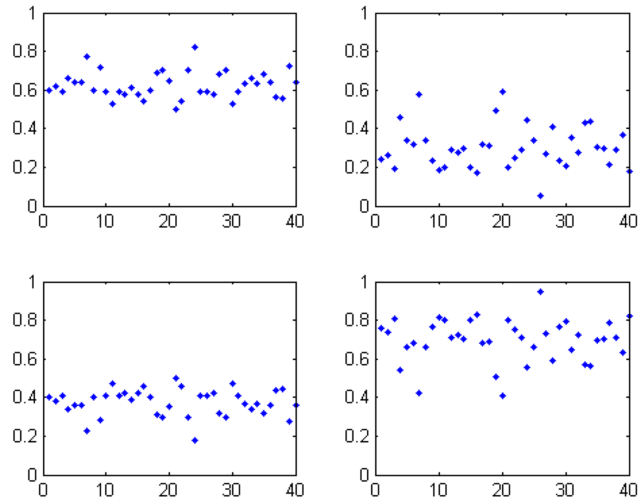


Figure 3.1: Intermediate Share of Each Sector by Country

Note : (i,j) figure means the intermediate share of i sector good in j sector by country ; 1=Tradable Sector / 2= Nontradable sector

the analysis, I focus on the interaction between tradable and nontradable goods. In the Appendix B, there is table 3 that shows the categorization method employed.

Figure 1 shows the intermediate share of a sector i good in j sector by country (The figure of finer classification is in Appendix B). The x-axis indicates the country index and the y-axis represents the share of intermediate inputs used to produce the corresponding goods. For example, the first point in the (1,2) figure means that non-tradable sector in the 1st country spends 20% of its intermediate spending to buy the goods from tradable sector. Clearly, if ω_{ij} is same across country, each figure should exhibit a straight line.

Even though the data doesn't display a perfectly straight line, the intermediary spending share is similar across countries.[4]. Therefore, I will use the U.S as the benchmark case and assume the rest of world has the same input -output linkage as the model assume. To calculate gross expenditure ratio by sector, ν_i, we need personal expenditure ratio, β_i, which comes from BEA industry data.

[4]Outlier countries are Bulgaria, China, India, Indonesia

Parameter/variable	Value
θ	4.34
η	5.30
α	0.45
τ	1.47
T_t^*	0.08
T_{nt}^*	0.09
F	$4.6 \cdot 10^{-6}$
L^*/L	25
ω_{TT}	0.7
ω_{NN}	0.8
w^*/w	4.09

Table 3.2: Benchmark Parameters

Table 2 shows the calibrated benchmark parameters. The elasticity of substitution is assumed to be 5, which is a commonly used value in the literature. To correspond to Zipf's law, the firm heterogeneity parameter, θ is set to 4.34. I adopt the parameter the fixed cost from Gaubert and Itskhoki (2016). α is chosen to match GDP to gross output ratio in the U.S. Labor is given by the population ratio of world against the U.S. from the World Bank data set. Finally, with technology normalized in the home country, the foreign technology level and trade cost in each sector are calibrated to match the foreign share of the U.S coming from U.S. International Trade Data of U.S Census Bureau. Finally, ω_{TT} gives the intermediate share of tradable goods from tradable sector. The solution procedure with which I estimate trade costs and do counter-factual is given by

Solution Procedure

1. Normalize W as 1 and guess W^*
2. Given W^*, solve for price of home and foreign to equate
   1) Sectoral equilibrium equation (3.17)
   2) Labor market clearing condition equation (3.24)[5]
   3) Expenditure ratio equation (3.23)

[5] This equation can be directly put into (3.17)

3. With foreign share driven from the 2nd step procedure, calculate $\sum_{i=1}^{S} \Phi_i E_i$.
4. Update W^* until trade balance becomes 0.

3.4 Counterfactuals

In this section, I measure the welfare gain in the benchmark when going from autarky to free trade. I also measure the welfare gain from various counter-factuals to compare the standard theory with the benchmark and analyze the role of input-output linkage, especially the interaction between tradable and non-tradable goods. I also measure the welfare gain going from autarky to trade at a finer, sector-level analysis. The result is exactly same as the reported benchmark result. To simplify analysis of linkage network as counterfactuals, I categorize into just two sectors.

Figure 3.2 illustrates possible network linkages we can use in counter-factual analysis. The arrow direction shows the flow of consumption or intermediate goods. The first diagram depicts the features of standard theory, with no channel between different sectors on supply side. The only way they can interact is through consumers. When it comes to welfare gain, part of the problem of the standard theory can be traced to this omission. In a partial equilibrium sense, a favorable environment in one sector is unfavorable to the other sector because wage payments become more expensive, which canceled the initially large gains from trade to some extent.

The benchmark diagram adds network linkages. There is not only input-output linkage within sectors, but also across sectors. On the other hand, in the third diagram, there is no linkage across sectors as in the standard theory, but firms interact within their own sector. Finally, the last diagram is when the linkage dependence is biased toward tradable sector – causing the cascading effect mentioned in Acemoglu et al. (2012). In the paper, they measures the impact of each industries' network effect with out-degree defined by

$$d_i = \sum_{j=1}^{S} \omega_{ij} \tag{3.25}$$

Therefore, the last figure in figure 3.2 is when the tradable good has the highest possible (out) degrees – higher dependence on tradable goods.

	Benchmark	Standard	τ_T=1	$\tau_{NT} = \tau_T$	Smallest Outdegree	Largest Outdegree	No Network Linkage
P^A/P	0.0429	0.0211	0.1768	0.1944	0.0161	0.1213	0.0330

Table 3.3: Welfare Gains from Trade

To begin my analysis of welfare gains from trade, I compare the gains from my model to the standard model. Figure 3 shows the welfare gain as the trade cost decreases. Clearly, welfare gain increases as trade costs decreases. Moreover, the presence of network effects enhances the welfare gains through the linkage channel, which confirms the results of previous work.

Further, I impose other network structures as a counter-factual analysis to understand the role of network effects on welfare gains. The last 3 columns of table 3 correspond to this exercise. 'Largest out-degree' and 'No network linkage' are the cases corresponding to the last 2 diagrams in figure 3.2 as mentioned above. 'Smallest out-degree' is the opposite case of 'Largest out-degree.'

Clearly, the welfare gain is the biggest when the structure of the economy depends more on tradable goods. On the other hand, the opposite case is even smaller than the standard theory. This implies that the presence of intermediate inputs itself is not enough for high welfare gains. Higher welfare gains should come along together with overall improvement in the modeling of the non-tradable sector through the network linkages.

A disappointing result is that although the measured data reveals the presence of interaction between tradable sectors and non-tradable sectors, it's not high enough to produce high welfare gain from trade. ($\omega_{NT} = 0.2$ and $\omega_{TN} = 0.3$) The fourth column of the table 3 shows that the welfare gain becomes high only if the non-tradable sector were to become tradable. Surprisingly, the welfare gain is even higher in this case than when there is no trade cost for tradable goods. Therefore, it appears that any further gains will need to come from improving our understanding of the ways openness can affect the non-tradable sector.

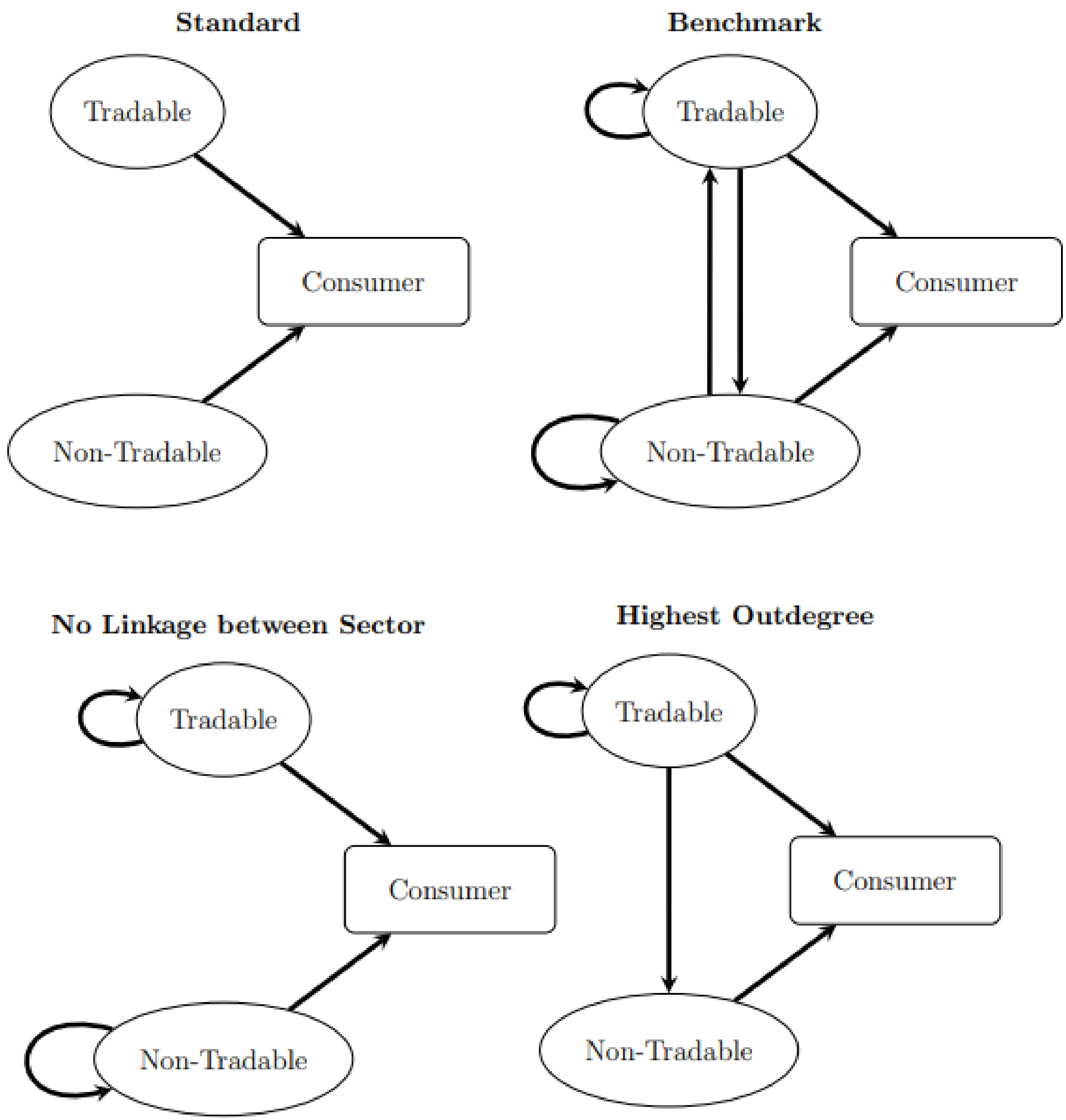


Figure 3.2: Possible network linkages

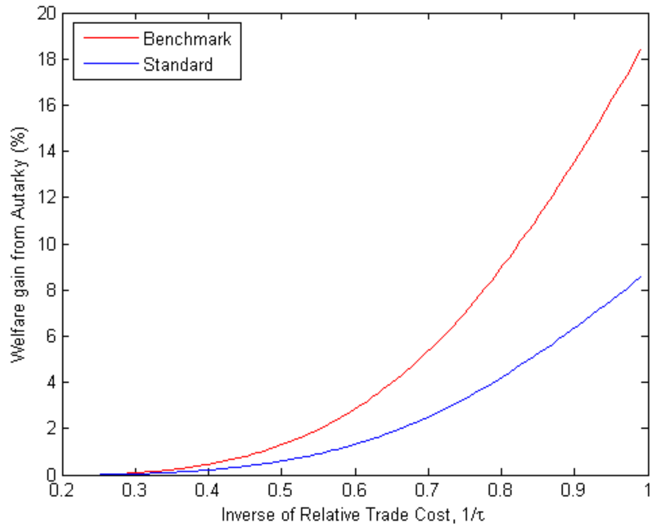


Figure 3.3: Comparison between the Model and the Standard Melitz

3.5 Concluding Remark

This paper develops a multi-sector model based on Melitz (2003) with input-output linkages. With this framework, we can analyze how firms interact with each other across sectors and countries. When goods are delivered to other countries, it initiates a positive chain reaction, increasing the size of welfare gains through the network channel, a feature that is excluded from many standard models.

The results show a modest improvement in welfare gains from trade. Welfare gains become larger when there is a linkage channel from tradable to nontradable goods. However, the results are not as strong as expected due to the relatively small importance of tradable goods for an economy. To get higher welfare gains, we likely need to consider other channels to bring about the improvement in non-traded goods as well as tradable goods. Still, sectoral linkages appear to be a promising path forward and there are probably more indirect ways in which these linkages affect the production of non-tradable goods in an economy that opens to trade. Future research should attempt to explore more complicated models that can incorporate features like allowing for different production and network systems across countries. Generalizing the model by including CES technology and estimating parameters directly could also add precision.

3.6 Appendix. A

3.6.1 Proof of Sectoral Equilibrium

Rewrite the profit equation (14)

$$\begin{aligned}\pi_i(u) = &\Big[(\frac{\eta}{\eta-1})^{1-\eta}(\frac{mc_i}{P_i})^{1-\eta}\varphi_i(u)^{\eta-1}E_i - WF\Big]^+ \\ &+ \Big[(\frac{\eta}{\eta-1})^{1-\eta}(\frac{mc_i}{P_i})^{1-\eta}\varphi_i(u)^{\eta-1}E_i^* - W^*F^*\Big]^+\end{aligned} \tag{3.26}$$

Together with the foreign counterpart of the equation, we can get cutoff level of operating domestic firm($\varphi_{i,h}$), exporting domestic firm($\varphi_{i,h}$), operating foreign firm($\varphi^*_{i,f}$), and exporting foreign firm($\varphi^*_{i,h}$), respectively.

$$\varphi_{i,h} = \frac{\eta}{\eta-1}\frac{mc_i}{P_i}\Big(\frac{\eta WF}{E_i}\Big)^{\frac{1}{\eta-1}} \tag{3.27}$$

$$\varphi_{i,f} = \frac{\eta}{\eta-1}\frac{mc_i}{P_i^*}\Big(\frac{\eta W^*F^*}{E_i^*}\Big)^{\frac{1}{\eta-1}} \tag{3.28}$$

$$\varphi^*_{i,h} = \frac{\eta}{\eta-1}\frac{mc_i^*}{P_i}\Big(\frac{\eta WF}{E_i}\Big)^{\frac{1}{\eta-1}} \tag{3.29}$$

$$\varphi^*_{i,f} = \frac{\eta}{\eta-1}\frac{mc_i^*}{P_i^*}\Big(\frac{\eta W^*F^*}{E_i^*}\Big)^{\frac{1}{\eta-1}} \tag{3.30}$$

The definition of domestic price index (6) can be decomposed into

$$P_i = \Big[\int_{u\in M_i} p_i(u)^{1-\eta}du + \int_{u\in M_i^f} p_i(u)^{1-\eta}du\Big]^{\frac{1}{1-\eta}} \tag{3.31}$$

where M_i is the set of continuing domestic firms and M_i^f is the set of foreign firms to export to the domestic market. Given exogeneous mass M_i together with cutoff level (3.27)-(3.30) and a Pareto distribution (3.7),

$$P_i = \frac{\eta}{\eta - 1} mc_i(\frac{\theta T_i}{\theta - \eta + 1})^{\frac{1}{1-\eta}} (\varphi_{i,h})^{\frac{\eta-1-\theta}{1-\eta}} \Big[1 + (mc_i/\tau_i mc_i^*)^{\theta} T_i^*/T_i\Big]^{\frac{1}{1-\eta}} \tag{3.32}$$

Plug this value back into the cutoff level (3.27) - (3.28) and solve for the cutoff level

$$\varphi_{i,h} = \Big(\frac{\theta T_i}{\theta - \eta + 1}\Big)^{\frac{1}{\theta}} (1 - \Phi_i)^{-\frac{1}{\theta}} \Big(\frac{\eta W F}{E_i}\Big)^{\frac{1}{\theta}} \tag{3.33}$$

$$\varphi_{i,f} = \Big(\frac{\theta T_i^*}{\theta - \eta + 1}\Big)^{\frac{1}{\theta}} (1 - \Phi_i^*)^{-\frac{1}{\theta}} \Big(\frac{\eta W^* F^*}{E_i^*}\Big)^{\frac{1}{\theta}} \tag{3.34}$$

$$\varphi_{i,h}^* = \frac{mc_i}{\tau_i mc_i^*} \varphi_{i,h} \tag{3.35}$$

Finally combined with the price index (3.32), we can get the desired result (3.17)

$$P_i = \frac{\eta}{\eta - 1} mc_i \Big(\frac{\theta T_i}{\theta - \eta + 1}\Big)^{-\frac{1}{\theta}} (1 - \Phi_i)^{\frac{1}{\theta}} \Big(\frac{\eta W F}{E_i}\Big)^{\frac{\theta-\eta+1}{\theta(\eta-1)}} \tag{3.36}$$

3.6.2 Proof of Sectoral Expenditure Share

Rewrite the gross expenditure equation, (3.21)

$$E_i = \beta_i (WL + \Pi) + \sum_{j=1}^{S} P_i X_{ij} \tag{3.37}$$

from the value of the table 1, we can see the equation becomes

$$E_i = \beta_i \frac{1 + \alpha(\eta - 1)}{\eta} \sum_{j=1}^{S} ((1 - \Phi_j) E_j + \Phi_j E_j^*) + (1 - \alpha) \frac{\eta - 1}{\eta} \sum_{j=1}^{S} \omega_{ij} ((1 - \Phi_j) E_j + \Phi_j^* E_j^*) \tag{3.38}$$

Define E such that i th component is E_i and let's define $NX_j = \Phi_j^* E_j^* - \Phi_j E_j$ Then, the system of equation becomes[6]

$$E = \beta \frac{1+\alpha(\eta-1)}{\eta} \mathbf{1}' E + \frac{1+\alpha(\eta-1)}{\eta} \mathbf{1}' NX + (1-\alpha)\frac{\eta-1}{\eta}\Omega E + (1-\alpha)\frac{\eta-1}{\eta}\Omega NX \quad (3.39)$$

where $\mathbf{1}'$ is S×1 vector whose all element is 1. Note that by the trade balance equation, $\mathbf{1}' NX = 0$.

Define $e \equiv E/1'E, nx \equiv NX/1'E$

$$e = \frac{1+\alpha(\eta-1)}{\eta}\beta + (1-\alpha)\frac{\eta-1}{\eta}\Omega e + (1-\alpha)\frac{\eta-1}{\eta}\Omega nx \quad (3.40)$$

Therefore,

$$e = \frac{1+\alpha(\eta-1)}{\eta}[I - (1-\alpha)\frac{\eta-1}{\eta}\Omega]^{-1}\beta + \{[I - (1-\alpha)\frac{\eta-1}{\eta}\Omega]^{-1} - I\} nx \quad (3.41)$$

Further, define $\nu \equiv \frac{1+\alpha(\eta)}{\eta-1}[I - (1-\alpha)\frac{\eta-1}{\eta}\Omega]^{-1}\beta$ and $\Gamma \equiv [I - (1-\alpha)\frac{\eta}{\eta-1}\Omega]^{-1} - I$ Then,

$$e = v + \Gamma nx \quad (3.42)$$

On the other hand, we can express the net export as the matrix form that follows the next equation.

$$NX = \tilde{\Phi}^* E^* - \tilde{\Phi} E \quad (3.43)$$

where the ith diagonal element of $\tilde{\Phi}$ is Φ_i, elsewhere 0. Therefore,

$$nx = \tilde{\Phi}^* e^* \frac{\mathbf{1}' E^*}{\mathbf{1}' E} - \tilde{\Phi} e \quad (3.44)$$

[6] Abusing notation, E is now the vector of expenditure

With labor market clearing condition, (3.24), the total expenditure ratio is equal to wage ratio between 2 country.

$$\frac{\mathbf{1}'E^*}{\mathbf{1}'E} = \frac{W^*L^*}{WL} \tag{3.45}$$

After some matrix manipulation, equation (3.42), (3.44) and it's counterpart together with (3.45) yields.

$$e = v + \Big\{I + \Gamma\Big[I + \tilde{\Phi}^*\Gamma\Big]^{-1}\tilde{\Phi}\Big\}^{-1}\Gamma[I + \tilde{\Phi}^*\Gamma\Big]^{-1}\Big\{\tilde{\Phi}^*\frac{W^*L^*}{WL} - \tilde{\Phi}\Big\}\nu \tag{3.46}$$

Simplifying further,

$$e = \nu + \Big\{\tilde{\Phi}^* + \Gamma^{-1} + \tilde{\Phi}\Big\}^{-1}\Big\{\tilde{\Phi}^*\frac{W^*L^*}{WL} - \tilde{\Phi}\Big\}\nu \tag{3.47}$$

tabular

3.7 Appendix B

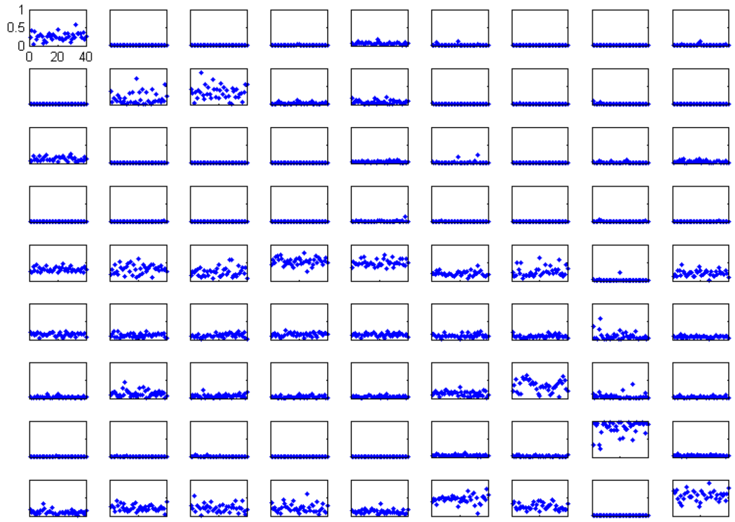


Figure 3.4: Intermediary Share of Each Sector by Country

Note : (i,j) figure means the intermediary share of i sector good in j sector by country x axis indicates the index of country; The list of industries : 'Agriculture', 'Mining', 'Utilities', 'Construction', 'Manufacturing', 'Wholesale - Retail trade', 'Transportation', 'Information', 'Services'

	NAICS	WIOD
Tradable	Agriculture	Agriculture, Hunting, Forestry and Fishing
	Mining	Mining and Quarrying
	Manufacturing	Food, Beverages and Tobacco Textiles and Textile Products Leather, Leather and Footwear Wood and Products of Wood and Cork Pulp, Paper, Printing and Publishing Coke, Refined Petroleum and Nuclear Fuel Chemicals and Chemical Products Rubber and Plastics Other Non-Metallic Mineral Basic Metals and Fabricated Metal Machinery, Nec Electrical and Optical Equipment Transport Equipment Manufacturing, Nec; Recycling
Non-tradable	Utilities	Electricity, Gas and Water Supply
	Construction	Construction
	Wholesale & Retail	Sale, Repair of Motor Vehicles and Motorcycles Wholesale Trade and Commission Trade Retail Trade
	Transport	Inland Transport Water Transport Air Transport Other transport
	Information	Post and Telecommunications
	Services	Hotels and Restaurants Financial Intermediation Real Estate Activities Renting of M&Eq and Other Business Activities Education Health and Social Work Other Community, Social and Personal Services Private Households with Employed Persons

Table 3.4: Industry Classification used in Figure 1 and Figure 3

Index Number	Country's name	WIOD code
1	Australia	AUS
2	Austria	AUT
3	Belgium	Bel
4	Bulgaria	BGR
5	Brazil	BRA
6	Canada	CAN
7	China	CHN
8	Cyprus	CYP
9	Czech Republic	CZE
10	Germany	DEU
11	Denmark	DNK
12	Spain	ESP
13	Estonia	EST
14	Finland	Fin
15	France	FRA
16	United Kingdom	GBR
18	Hungary	HUN
19	India	IDN
20	Indonesia	IND
21	Ireland	IRL
22	Italy	ITA
23	Japan	JPN
24	Korea	KOR
25	Lithuania	LTU
26	Luxembourg	LUX
27	Latvia	LVA
28	Mexico	MEX
29	Malta	MLT
30	Netherlands	NLD
31	Poland	POL
32	Portugal	PRT
33	Romania	ROM
34	Russia	RUS
35	Slovakia	SVK
36	Slovenia	SVN
37	Sweden	SWE
38	Turkey	TUR
39	Taiwan	TWN
40	United States	USA

Table 3.5: Country list

BIBLIOGRAPHY

Acemoglu, D. and Cao, D. (2015). Innovation by entrants and incumbents. *Journal of Economic Theory*, 157:255–294.

Acemoglu, D., Carvalho, V. M., Ozdaglar, A., and Tahbaz-Salehi, A. (2012). The Network Origins of Aggregate Fluctuations. *Econometrica*, 80(5):1977–2016.

Achdou, Y., Han, J., Lasry, J.-M., Lions, P.-L., and Moll, B. (2017). Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach. *NBER Working Paper*.

Aghion, P. and Bolton, P. (1997). A Theory of Trickle-Down Growth and Development. *The Review of Economic Studies*, 64(2):151.

Aiyagari (1994). Uninsured Idiosyncratic Risk and Aggregate Saving. 109(3):659–684.

Aiyagari, S. (1994). Uninsured Idiosyncratic Risk and Aggregate Saving. *The Quarterly Journal of Economics*, 109(3):659–684.

Anzoategui, D., Comin, D., Gertler, M., and Martinez, J. (2019). Endogenous technology adoption and R & D as sources of business cycle persistence. *American Economic Journal: Macroeconomics*, 11(3):67–110.

Arkolakis, C., Costinot, A., Rodríguez-Clare, A., Angeletos, M., Antràs, P., Atkeson, A., Burstein, A., Donaldson, D., Eden, M., Gopinath, G., Grossman, G., Klenow, P., Komunjer, I., Kortum, S., Maggi, G., Mcgrattan, E., Ruhl, K., Stokey, N., Tybout, J., Vogel, J., and Yi, K.-M. (2012). New Trade Models, Same Old Gains? *American Economic Review*, 102(1):94–130.

Axtell, R. L. (2001). Zipf distribution of U.S. firm sizes. *Science*, 293(5536):1818–1820.

Balistreri, E. J., Hillberry, R. H., and Rutherford, T. F. (2011). Structural estimation and solution of international trade models with heterogeneous firms. *Journal of International Economics*, 83(2):95–108.

Baqaee, D. R. (2014). Cascading Failures in Production Networks. pages 1–60.

Bhandari, A. and McGrattan, E. R. (2018). Sweat Equity in U.S. Private Business Staff Report 560 Revised March 2018. *Federal Reserve Bank of Minneapolis Staff Report*.

Boyd, J. H. and Prescott, E. C. (1986). Financial intermediary-coalitions. *Journal of Economic Theory*, 38(2):211–232.

Buera, F. J. and Shin, Y. (2013). Financial frictions and the persistence of history: A quantitative exploration. *Journal of Political Economy*, 121(2):221–272.

Caliendo, L. and Parro, F. (2012). Estimates of the trade and welfare effects of NAFTA. *Review of Economic Studies*, 82(1):1–44.

Cole, D. C. and Park, Y. (1983). Financial development in Korea, 1945-1978. *Harvard University Asia Center*, 106(1st ed).

Di Giovanni, J., Levnchenko, A. A., and Mejean, I. (2014). Firms, Destinations, and Aggregate Fluctuations. *Econometrica*, 82(4):1303–1340.

Durlauf, S. (1993). Nonergodic Economic Growth. *The Review of Economic Studies*, 60(2):349 366.

Easterly, W. and Levine, R. (2001). It ' s Not Factor Accumulation : Stylized Facts and Growth Models. *The World bank economic Review*, 15(2):177–219.

Gabaix, X. (1999). Zipf ' s Law and the Growth of Cities. *American Economic Review*, 89(2):129 − 132.

Gabaix, X. (2011). The Granular Origins of Aggregate Fluctuations. *Econometrica*, 79(3):733–772.

Gaubert, C. and Itskhoki, O. (2016). Granular Comparative Advantage. *Working Paper*.

Gourinchas, P. O. and Jeanne, O. (2013). Capital flows to developing countries: The allocation puzzle. *Review of Economic Studies*, 80(4):1484–1515.

Grossman, G. M. and Helpman, E. (1991). Quality Ladders in the Theory of Growth. *The Review of Economic Studies*, 58(1):43.

Hsieh, C.-T. and Klenow, P. J. (2009). Misallocation and Manufacturing TFP in China and India. *The Quarterly Journal of Economics*, 124(4):1403–1448.

Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, 17(5-6):953–969.

Hulten, C. (1978). Growth Accounting with Intermediate Inputs. *The Review of Economic Studies*, 45(3):511–518.

Johnson, R. C. (2014). Trade in intermediate inputs and business cycle comovement. *American Economic Journal: Macroeconomics*, 6(4):39–83.

Jones, C. I. (2011). Intermediate Goods and Weak Links in the Theory of Economic Development. *American Economic Journal: Macroeconomics*, 3(2):1–28.

King, R. G. and Levine, R. (1993). Finance and Growth : Schumpeter Might be Right. *The Quarterly Journal of Economics*, 108(3):717–737.

Klenow, P. J. and Rodríguez-Clare, A. (1997). The Neoclassical Revival in Growth Economics: Has It Gone Too Far? *NBER Macroeconomics Annual*, 12(1997):62–103.

Laeven, L., Levine, R., and Michalopoulos, S. (2015). Financial innovation and endogenous growth. *Journal of Financial Intermediation*, 24(1):1–24.

Long, J. and Plosser, C. (1983). Real Business Cycles. *Journal of Economic Literature*, 91(4):39–69.

López, R. A. (2005). Trade and growth: Reconciling the macroeconomic and microeconomic evidence. *Journal of Economic Surveys*, 19(4):623–648.

Melitz, M. J. . (2003). The Impact of Trade on Intraindustry Reallocation and Aggregate Industry Productivity: A Comment. *Econometrica*, 71(6):1695–1725.

Midrigan, V. and Xu, D. Y. (2014). Finance and misallocation: Evidence from plant-level data. *American Economic Review*, 104(2):422–458.

Mino, K. (2015). A simple model of endogenous growth with financial frictions and firm heterogeneity. *Economics Letters*, 127:20–23.

Moll, B. (2014). Productivity losses from financial frictions: Can self-financing undo capital misallocation? *American Economic Review*, 104(10):3186–3221.

Newman, A. F. and Banerjee, A. V. (1993). Occupational Choice and the Process of Development Author. *Journal of Political Economy*, 101(2):274–298.

Ossa, R. (2015). Why trade matters after all. *Journal of International Economics*, 97(2):266–277.

Pack, H. and Nelson, R. R. (1997). The Asian Miracle and Modern Growth Theory. *World Bank Policy Research Working Paper*.

Prescott, E. C. and Parente, S. L. (1994). Barriers to Technology Adoption and Development. *Journal of Political Economy*, 102(2):298–321.

Restuccia, D. and Rogerson, R. (2008). Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic Dynamics*, 11(4):707–720.

Romer, P. M. (1990). Endogenous Technological Change. *Journal of Political Economy*, 98(5).

Schumpeter, J. A. (1911). The theory of economic development. *Harvard University Press*.

Song, Z., Storesletten, K., and Zilibotti, F. (2011). Growing like China. *American Economic Review*, 101(1):196–233.

Stiglitz, J. E. and Yusuf, S. (2001). *Rethinking the East Asian Miracle*.

Young, A. (1995). The Tyranny of Numbers : Confronting the Statistical Realities of the East Asian Growth Experience. *The Quarterly Journal of Economics*, 110(3):641–680.